

**TECHNICAL UNIVERSITY OF CONSTRUCTION BUCHAREST
FACULTY OF TECHNOLOGICAL EQUIPMENT**

RESEARCH REPORT NO. 2

**BIOMECHANICAL MODELS PROPOSED FOR THE STUDY
OF THE ELBOW AND KNEE JOINTS UNDER THE ACTION
OF VIBRATION**

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**BUCHAREST
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1. INTRODUCTION

For the biomechanical modeling of the human body, especially the osteo-articular system, the laws and general principles of engineering are used. The model must be made so that the behavior of the original system can be determined, within certain precision limits.

In order to study a mechanical system subject to the action of vibrations, it is necessary to go through the following stages according to [1]: problem definition, physical modeling, mathematical modeling, dynamic study of the mathematical model, verification of the correctness of the model.

This research report presents biomechanical models of the human body and the hand-forearm-arm system, models of the elbow and knee joints.

1.1. Field of study

The osteo-articular system can be considered a deformable spatial structure, with a complex geometry, elastic properties and complicated loads.

An effective model can be created based on the known information about the osteo-articular system and the action of vibrations on the human organism. It must take into account:

- a) model geometry;
- b) resting and loading it - in conditions close to reality;
- c) the type of model – static, kinematic or dynamic.

Most bone elements have complicated geometric shapes. Their material is inhomogeneous and anisotropic. Their structure is complex: on the outside a hard and compact material, under which there is a spongy layer; the layers are arranged in such a way as to present maximum resistance of commonly encountered requests. The dimensions, shape, mechanical characteristics, elastic constants, physical constants of a bone differ greatly from one individual to another, depending on a multitude of factors, the most important of which are: age, sex, waist, profession, current physiological state, environmental conditions.

Dynamic modeling of the osteo-articular system is useful because :

- the circumstances in which fractures occur of the bones are in the vast majority of dynamic cases: fall, collapse, impact;
- exercising normal physiological actions and movements is essentially dynamic: walking, running, jumping ;
- the aim is to improve the prostheses used in surgery of the osteo-articular system;
- the effects of vibrations on the body are determined.

1.2. Purpose and objective

This report presents some biomechanical models of elbow and knee joints. The objective is to determine the pulsations and own frequencies using the Matlab program.

2. BIOMECHANIC MODELING OF THE HUMAN BODY AND A ITS JOINTS

The human body is a complex structure that involves many non-linearities, therefore creating a biomechanical model is difficult. Measuring movements and forces in the body is difficult to perform, and active voluntary and involuntary muscle control influences the process. The problems arise when obtaining the necessary empirical data and from the incomplete understanding of the body's modes of movement. However, there are numerous attempts to model a portion or the entire body.[9]

The laws of classical mechanics govern movement in the human body down to the cellular level. If atoms or molecules are investigated, these laws are no longer valid.

Gowaerts believes that "biomechanics is the science that deals with the study of the effects that mechanical forces have on the functional structure of man".

Due to the particular complexity of the human body, the biomechanical modeling of the processes that take place inside it is a difficult task, and the principles and rules of mechanical engineering can be successfully used. Thus, we can study equilibrium conditions under the action of applied forces, abstracting from motion using a static model. If we follow only the geometric aspect of the movement, without taking into account masses and forces, then the model is kinematic (only the notions of space and time intervene). In a dynamic model all the fundamental notions will come into play: space, time and mass. In any of these situations, the laws of motion of the component elements can be written.

The geometry of the model, planar or spatial, on a real scale or on various other scales, depending on the purpose pursued, has its importance. The nature of the material from which the model is made, the connections (geometric restrictions) and the applied loads must correspond as closely as possible to reality. [2]

In order to create an efficient biomechanical model, it is good not to neglect these properties of the human body:

Inhomogeneity and anisotropy. Mechanical characteristics and elastic constants can vary in the same organism.

Mobility. It introduces a large number of degrees of freedom. Large displacements of some component parts add nonlinearities.

Complexity. The loads that act simultaneously on the body as a whole or on some of its parts must be taken into account. The equations to be written are more complicated and require the use of specialized calculation programs.

In the operation of developing effective biomechanical models, with the aim of studying the dynamics of the human body subjected to the action of the vibrational field, the models must be as close as possible to the biological reality and highlight the mechanical movement with all its characteristics.[2]

The human body behaves like a deformable body, with particularly varied properties, making it necessary to make some simplifications, which led to the appearance of various models.

Real systems have been transformed into oscillating systems composed of masses that accumulate kinetic energy, springs that contain potential energy, and dampers responsible for energy dissipation.

The effects of vibrations on the human body turn out to be dynamic effects transmitted to the whole body. The locomotor system facilitates the transmission of movement from the source to the human body.[9]

The movement of a body represents its displacement action due to the interaction of all the forces acting on it. In the case of the human body, one can consider either the movement of the entire body, viewed as a unitary whole, or the differentiated movement of different segments or sets of segments of the body.

In relation to a given direction, in space, there are two elementary movements: translational movement and rotational movement. The movements of the human body are combinations of the two elementary movements relative to the axes of the considered geometric reference system.

Depending on the position of the chosen reference system, the movement can be *absolute*, when the reference system is fixed, and *relative*, when the reference system is, in turn, moving. Relative movement appears, for example, when considering the movement of one segment of the body with respect to another, relativity being with respect to the reference segment.

From a biomechanical point of view, material characteristics can be used to analyze a bone or muscle structure: *deformability* materiality and its *mechanical resistance*.

The deformability of the material is quantified by *the displacements* produced in the analyzed body. Deformations can be:

- *elastic* - when the deformations disappear with the cessation of the action of the forces that produced them, the body returning to its original shape;
- *plastic* - when the deformations are residual after the cessation of the action of the forces;
- *elasto - plastic* – when the deformations disappear only partially after the cessation of the action of the forces, and the displacements, which determine the size of a deformation, can be:
 - *linear (arrow)* ;
 - *angular (rotation)* .

The mechanical resistance of the material (*tension* or *unit effort*), at a point of it, is defined as the ratio between the value of the elemental force acting at that point and that of the related elemental area.

The possible elementary movements of a body are: translational movement and rotational movement. All other movements of the body, such as rototranslation, pivoting, plane-parallel movements, etc., are obtained by combining the elementary ones, considered in the plane or in space.

The movement of the body or kinematic segments is always related to a reference system. By *reference system* is understood as a non-deformable benchmark against which the positions of a material system are reported. Reference systems can be fixed or mobile, so that the movement related to a reference system considered fixed is called *absolute movement*, and the movement related to a mobile reference system is called *relative movement*.

In biomechanics, a mobile frame of reference usually originates at the body's center of gravity, moving with the body's motion. Such a reference system is also called *a relative reference system*. Figure 1 shows such a reference system.

The coordinates of a point related to this reference system are called relative. The origin of an absolute reference system is an arbitrary point, in general, but with the property of being fixed or considered fixed in space. The axes of this reference system are also fixed or considered fixed.

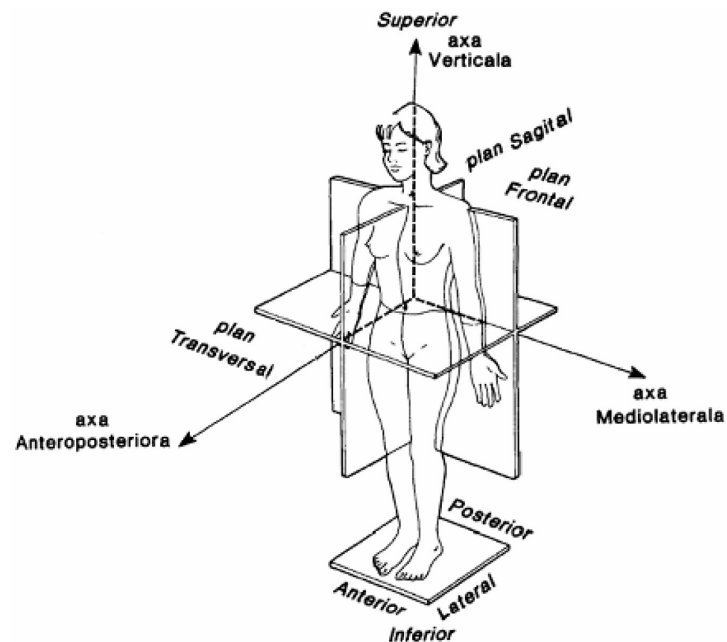


Figure 1

The separate representation of the reference planes is presented in figure 2. [5]

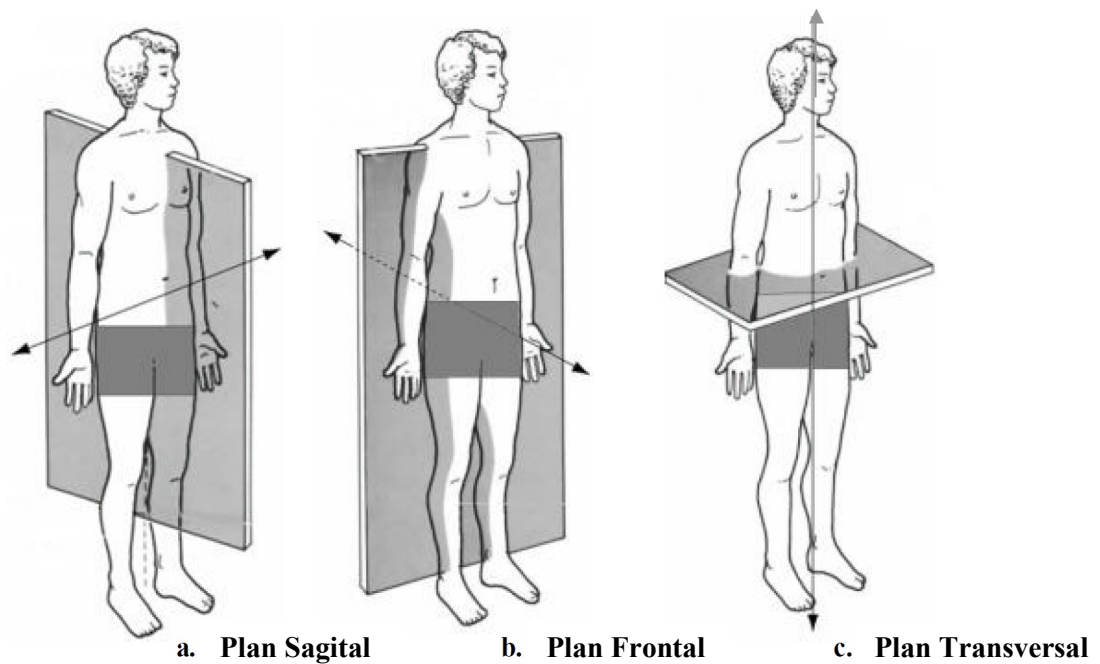


Figure 2

In biomechanics, the joints fulfill two well-defined main functions: they ensure the transfer of movement from one segment to another of the body, realizing the movement of the

skeleton, and they ensure the transmission of mechanical loads between the segments of the human body.

Articulation can be defined in several ways:

- from an anatomical point of view: "the assembly of soft elements that join two or more neighboring bones", "the connecting organs between the bones" or "the totality of the elements - represented by connective formations and muscles - by which the bones join each other"; according to this criterion, joints are considered including immobile ones, such as the joints of the bones of the cranial box;
- from a mechanical point of view: "the direct and movable connection between two kinematic elements, with the purpose of transmitting motion and force", "the system that restricts the freedom of movement of a point or system of material bodies" or "the area or areas of contact between two kinematic elements that determine the movement possibilities of the two elements";
- from a biomechanical point of view: "the connection or point of rotation between two or more bones" or "the anatomical assembly that ensures the transmission of movement and the transfer and dissipation of forces due to gravity or the muscular activity of the human body between two or more bones".[5]

Depending on the nature of the binding formations between the bones, with the appreciation of the shape of the bone ends, from a biomechanical point of view, the following categories of joints are distinguished:

- *fibrous joints* or *synarthrosis*, in which the connection is made by fibrous connective tissue, the bone ends having a complete congruence (joints by continuity), which practically leads to the almost complete disappearance of any movement; examples: skull joints (*sutures*), alveolodental joint;
- *cartilaginous joints* or *amphiarthrosis*, in which the connection between the bones is made by hyaline cartilage or by fibrocartilage, which allows some relative movement between the bones; examples: the bone pieces that make up the coxal bone, the joints between the vertebral bodies;
- *synovial joints* or *diarthrosis*, formed by joint surfaces, joint capsule and joint cavity, to which can be added, depending on the functional characteristics of each joint: ligaments, menisci, fat bodies, etc.; these joints ensure multiple and varied movements between the connecting bones; according to the shape of the bone ends, the following diarthrosis are differentiated:
 - planar joints (*arthrodes*), having mainly planar movements, where the articular surfaces are flat or slightly curved; example: tarsal joint;
 - spheroid joints (*enarthrosis*), where one bone head is spheroidal in shape, and the other, conjugated surface, is convex in shape; examples: hip, shoulder joints, etc.;
 - cylindroid joints (*trochlear* and *trochoid*), where the articular surfaces have conjugated cylindroid shapes; examples: elbow joint (trochlear), superior radio-ulnar joint (trochoid), etc.;
 - ellipsoid joints, where the articular surfaces have oval, ellipsoid shapes (*condyles*); examples: knee joints, neck, hand, etc.;
 - sellar joints (saddle-shaped), where the bone head has a concave shape in one direction and convex in another direction; example: calcaneo-cuboid joint.[5]

The basic structural elements specific to a synovial joint are shown in figure 3. The synovial articular cavity and the articular cartilages are what separate the articular bones, thus reducing the friction of the articular surfaces and absorbing the shocks produced during body movement. The joint capsule is the one that protects the joint by means of the fibrous connective tissue from which it is formed. Due to the fact that the structure of this capsule is almost identical to that of the ligaments, it is also called the ligament joint capsule. This capsule is lined on the

inside with a synovial membrane that has a series of important functions for joint movement. Thus, the synovial membrane is the one that produces the synovial fluid, thus also feeding the articular cartilage, it has the role of "cleaning" the joint through its macrophage function, it is a plasma filter, being highly vascularized and it represents a proprioceptor element due to the innervation which it contains. The joint capsule is not a continuous structure, forming synovial bags that facilitate tendomuscular sliding during joint movements.

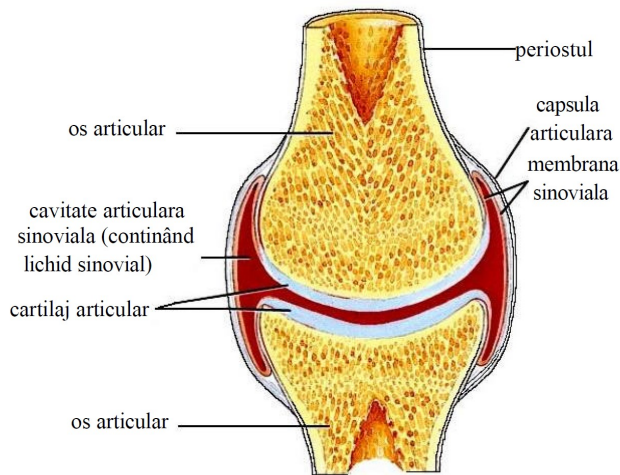


Figure 3 [5]

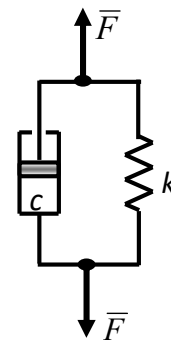


Figure 4

These joints, found in biomechanics, can allow *one, two* or *three* relative movements between the two bone bodies.[5]

The biomechanical functioning of a joint is influenced by two main characteristics:

- the anatomical shape of the articular surfaces, this being given by the type of movement (rolling, sliding or combined);
- the thickness of the cartilage layer which, together with the material properties and the applied load, determines the mechanical tension in the bone.[5]

The mathematical modeling of the joint investigates its working mode, the way the interaction of the elements affects its kinematics, the resistance to overload, breakage, fatigue and vibrations.

The human joint can be defined as a compression between two rigid bodies. Thus, the human skeleton can be represented as an articulated figure comprising several links that interact through the joints.

Of particular interest is the joint's ability to interactively change the orientation and position of the axis or center of rotation.[3]

Bone is considered a viscoelastic material because, with increasing stress, its strength and modulus of elasticity increase up to a certain limit. The physical model of such a material is represented in figure 4. They were denoted by k – the coefficient of elasticity, and by c – the damping coefficient, measured in [Ns/m].

The movement of the body is carried out under the action of the following internal forces: nervous impulse, muscle contraction and osteo-articular levers.

a) *Nerve impulse* - it is transmitted along the path of a reflex arc (receptors, afferent path, nerve center, efferent path and motor plate) to the muscle cell. The mechanisms underlying the movements are neuromuscular in nature.

b) *Muscle contraction* - a response reaction to stimulation by the nerve impulse. Muscle contraction is a manifestation related to the change in muscle elasticity. It manifests either as a strengthening of the muscle, or as a change in its strength and shape.

c) *Osteo-articular levers* . Nerve impulses produce muscle contractions, which in turn cause the displacement of bone segments at the level of muscle insertions, thus transforming chemical energy into mechanical energy. The bone segments on which the muscles act act like levers in physics. A rigid bar that can rotate about a fulcrum is considered a lever. The bones of the body have the axis of rotation in the joints, the active force is given by the muscles, and the resistance force is given by the weight of the body or its segments. The levers have the role of transmitting the movement, from the muscles and tendons to the resistive load, increasing its efficiency.

There are three types of levers, depending on the placement of the two forces (active and resistance) in relation to the support point:

Levers of the first degree - the levers at which the fulcrum is located between the two moments of force application and of resistance, both directed in the same direction. Examples: at the level of the joint between the skull and the vertebral column (atlanto-occipital), at the level of the coxo-femoral joint (in a sitting position). In the human body, all first degree levers have unequal arms, therefore the forces that balance them are also unequal. They are balance levers.

Second degree levers - levers that have the point of support at one end, force at the other end, and resistance in between. In the human body, this kind of levers is disputed, most authors admit that there is only one example, at the talocrural joint, in the position - standing on the tips. These are levers of force.

Grade III levers - levers that have the point of support at one end of the lever, resistance at the other end, and force between these. In the body humanly, this kind of leverage is very widespread. They act with loss of force and gain of displacement. Example: the elbow joint, where the support point is in the joint, the resistance at the other end (given by the weight of the forearm and hand), and the force is between them (given by the flexor muscles of the forearm on the arm).

These are gear levers.[6]

The external forces involved in making the movement are:

a) *the force of gravitational attraction* represents the affirmation of a universally valid law in nature. According to the law of universal attraction, the earth attracts bodies and at the same time is attracted by them. To overcome this force, a large amount of energy is required. The force of attraction of the earth acts on every molecule of the body. The sum of the attractive forces exerted on all the molecules of a body makes up the total gravitational attractive force acting on that body. It can be considered that a single force acts on a body, applied at a single point, called the center of gravity.

b) *body weight* always acts vertically, from top to bottom on the center of gravity of the body or segment. The value of this force is related to volume, length, density the moving segment or the number of segments engaged in movement. Practically, the value of this force is related to the mass of the moving segment.

c) *atmospheric pressure* it presses on the body with a variable intensity depending on the speed of travel. The action of the atmospheric pressure on the body is compensated by the internal pressure of the large cavities with values identical to those of the atmospheric pressure.

d) *the resistance of the environment* is manifested when the human body is in motion performing various activities in the open air or in water. It depends on the size of the frontal surface that the body opposes to the environment.

e) *inertia* it is the force that maintains the body's state of rest or movement.

f) *frictional force* it is proportional to the weight of the body sliding on a support surface and to the coefficient of friction.

Internal and external forces work together and influence each other. The entire human activity takes place with the help of these forces, in which the decisive role is played by the muscular contraction directed by the cerebral cortex.[6]

In order to be able to model the human body, which behaves like a deformable body, simplified biomechanical models are needed. Real systems thus became oscillating systems made up of springs (accumulate potential energy), shock absorbers (dissipate energy) and masses with the role of storing kinetic energy.

Among the models created over time for modeling the human body are:

Saint Venant's solid is a body in which deformations occur after stresses reach a certain threshold. The model is also called the plastic-rigid body (figure 5).

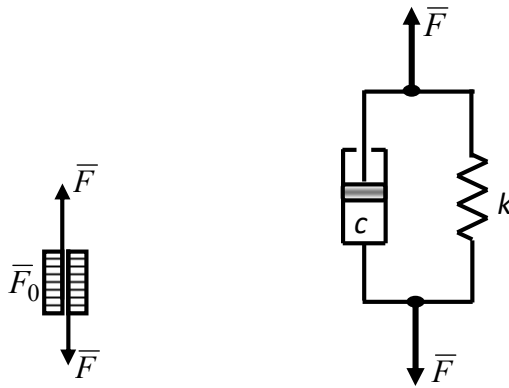


Figure 5

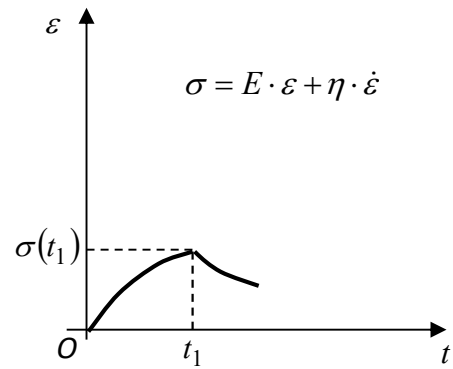


Figure 6

The model in which the tension is directly proportional to the deformation and the deformation speed is called *the Kelvin-Voigt body* (figure 6).

Connecting a spring in series with a damper results in a model that behaves differently in relation to the speed of application of the load. This model is known as *Maxwell's body* (figure7).

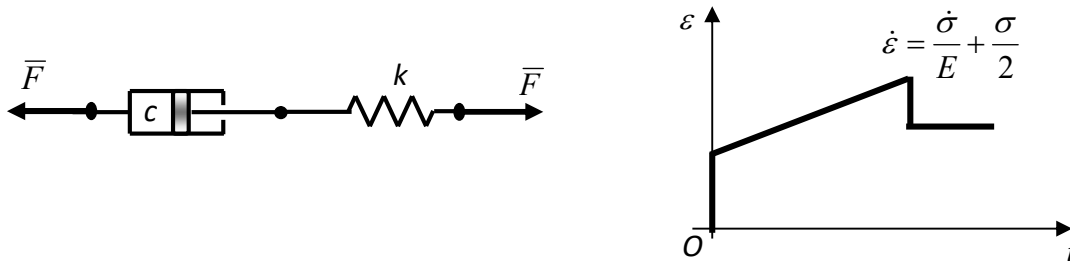


Figure 7

3. BIOMECHANICAL MODELS OF THE HAND – FOREARM – ARM SYSTEM AND OF THE HUMAN ELBOW JOINT

3.1. Biomechanical models of the hand-forearm-arm system

The first biomechanical model of the hand-forearm-arm system was made by Dieckmann in 1957. The purpose of the model was to determine the impedance and the frequency response function in the vertical direction. Attempts to develop biomechanical models, by combining simple models, existed both before and after this year. Figure 8 shows two models made by Kuhn (figure 8a) in 1953 and Dieckmann (figure 8b) in 1958.

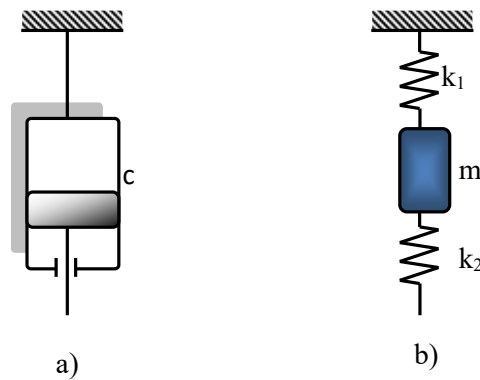


Figure 8

In 1984, Professor Magheți created a biodynamic model of this system, which highlights the dependence of the frequency response on the operator's working position. The model studies the complex movement of the hand-arm system (rotation, plane-parallel), not only translational movements (figure 9).

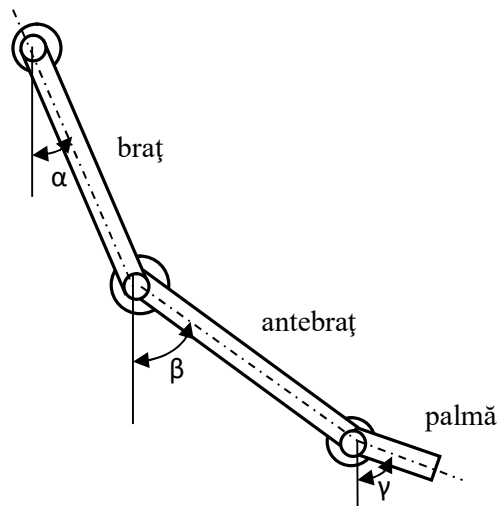


Figure 9

Walter Murel together with a group of collaborators makes the assumption that the bones of which the upper limb is composed can be modeled in the form of rigid cylinders, and the biomechanical model appears in 1996 (figure 10).

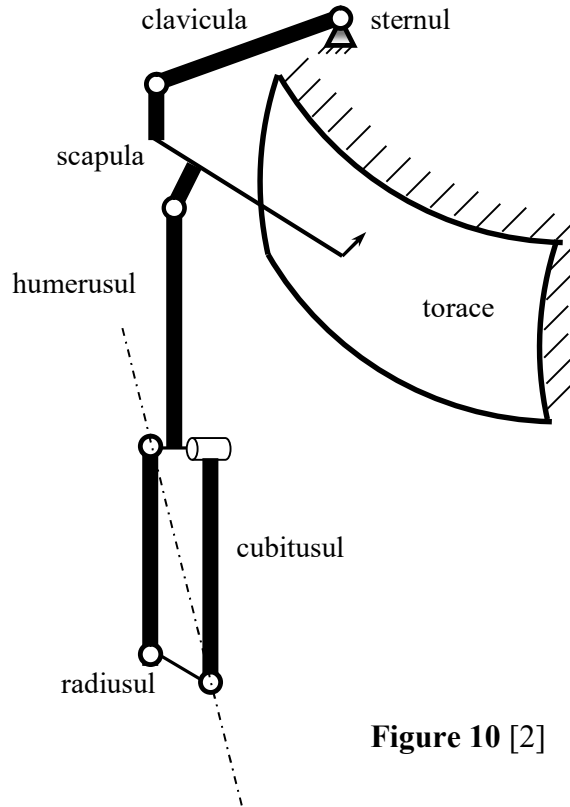


Figure 10 [2]

Alexandra Picu, in her doctoral thesis, analyzes the dynamic behavior of the hand-arm system under the action of vibrations, on a model with three degrees of freedom, assimilated to a triple pendulum, whose masses move in a vertical plane, with restrictions kinematics at the free end (figure 11).[2]

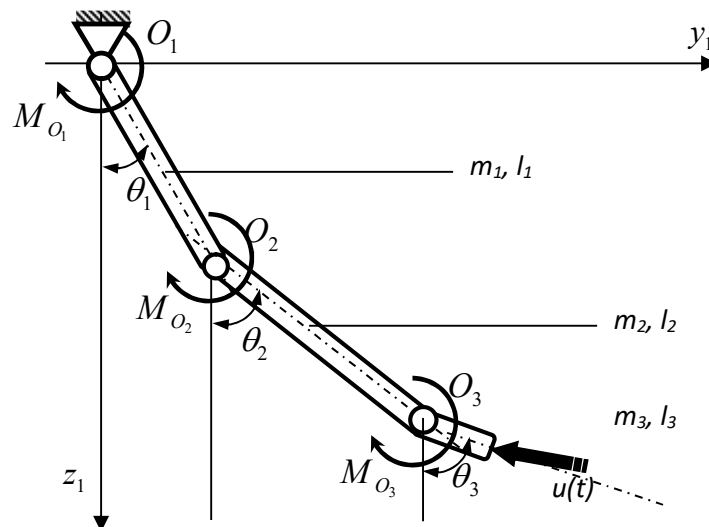


Figure 11

In figure 11, $M_{O_1}, M_{O_2}, M_{O_3}$ represents the total resultant moments, and $u(t)$ has the form: $u(t) = u_o \cdot \sin \omega t$. Using Lagrange's equations of the II-a type and making successive substitutions, a system of three equations is obtained that can be written in matrix form as follows:

$$[J] \cdot [\cos([\Delta\theta])] \cdot [\ddot{\theta}] + [C] \cdot [\dot{\theta}] + [K] \cdot [\theta] = -[S] \cdot g \cdot [\sin([\theta])] - [J] \cdot [\sin([\Delta\theta])] \cdot [\dot{\theta}^2] + [Ex] \cdot \ddot{u},$$

in which,

- $[J]$ is the inertia matrix of the system;
- $[K]$ is the stiffness matrix of the system;
- $[C]$ is the matrix of viscous dissipation coefficients;
- $[S]$ is the influence matrix;
- $[Ex]$ is the excitation vector.

Due to the complexity and non-linearities, the solution of this system was achieved by numerical methods.

A group of researchers from the University of Guelph in Canada, consisting of Hussein Abdullah, Cole Tarry, Rahul Datta, Gauri Mittal and Mohamed Abderrahim, published, in 2007, an article presenting a dynamic biomechanical model used in the evaluation and monitoring of upper limb therapy. The shoulder joint is represented with three degrees of freedom, and the elbow joint with two degrees of freedom (figure 12).

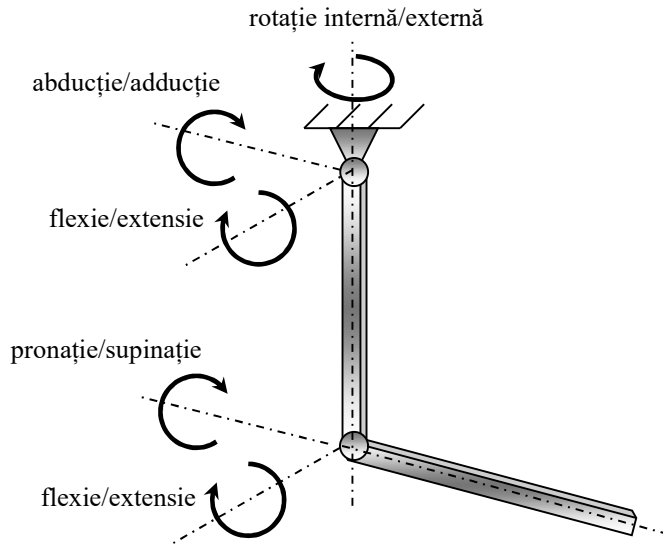


Figure 12

A model with three degrees of freedom of the human finger was made and presented as part of his doctoral thesis by Mr. Radu Panaitescu-Liess, member of the Department of Mechanical Technology at UTCB. The bones of the finger, called phalanges, are short, three each for each finger except the thumb which has two. The first phalanx - phalanx proximalis - articulates with the metacarpal bone. The second phalanx - the medial phalanx - is attached by means of the interphalangeal joints. Finally, the third phalanx - phalanx distalis - is attached at one end to the interphalangeal joint, the other end being free and sharp (figure 13).[2]

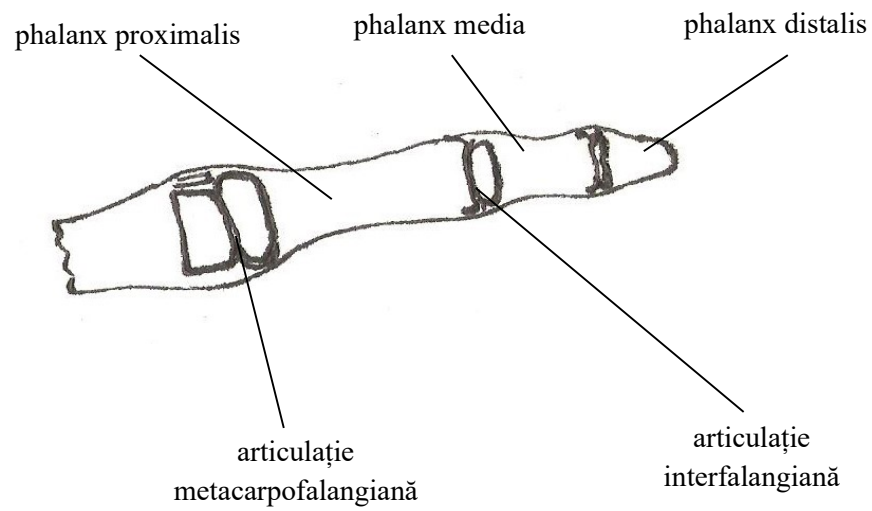


Figure 13

The movement can be studied in the yOz plane (vertical plane), the three phalanges being assimilated with three bars of mass m_i and lengths l_i , connected by cylindrical joints in which act springs with elastic constants k_{ri} and shock absorbers with damping coefficients c_{ri} (figure 14).

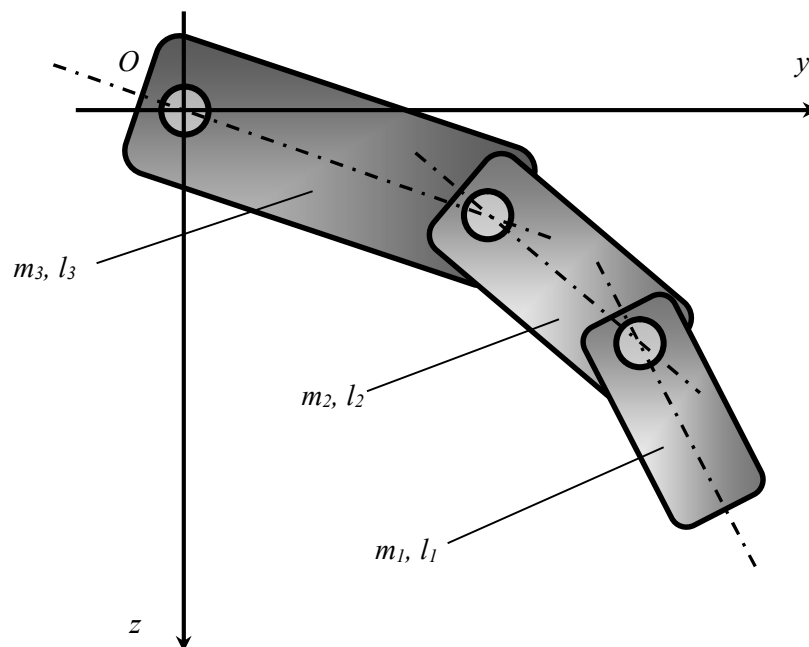


Figure 14

Panaiteescu-Liess presented a model with 8 degrees of freedom of the hand-forearm-arm system and calculated, with the help of the Matlab program, its own pulsations and frequencies (figure 15). System dampings have been neglected.

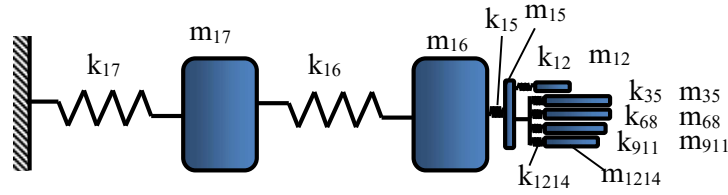


Figure 15

To determine the pulsations and natural frequencies, the experimental data for masses and stiffnesses from [21] were used (tables 1 and 2).

Table 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
M [kg]	0.014	0.023	0.008	0.012	0.026	0.01	0.015	0.029	0.007	0.01	0.023	0.006	0.007	0.016	0.213
k [kN/m]	11.59	5,527	10.89	9,713	5,431	10.94	9,661	5,169	9,146	8.19	4,804	2,994	7,761	4,675	15.43

Table 2

	15	16	17
M [kg]	0.213	0.016	0.213
K [kN/m]	15.43	4,675	15.43

The inertia [M] and stiffness [K] matrices are of the form:

$$[M] = \begin{bmatrix} m_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{68} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{911} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{1214} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{16} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{17} \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_{12} & 0 & 0 & 0 & 0 & -2 \cdot k_{12} & 0 & 0 \\ 0 & k_{35} & 0 & 0 & 0 & -k_{35}/2 & 0 & 0 \\ 0 & 0 & k_{68} & 0 & 0 & -k_{68}/2 & 0 & 0 \\ 0 & 0 & 0 & k_{911} & 0 & -k_{911}/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{1214} & -k_{1214}/2 & 0 & 0 \\ -2 \cdot k_{12} & -k_{35}/2 & -k_{68}/2 & -k_{911}/2 & -k_{1214}/2 & k_{15} + 4 \cdot k_{12} + 0,25 \cdot (k_{35} + k_{68} + k_{911} + k_{1214}) & k_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_{15} & k_{15} + k_{16} & -k_{16} \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_{16} & k_{16} + k_{17} \end{bmatrix}$$

where:

$$\begin{aligned}m_{12} &= m_1 + m_2 \\m_{35} &= m_3 + m_4 + m_5 \\m_{68} &= m_6 + m_7 + m_8 \\m_{911} &= m_9 + m_{10} + m_{11} \\m_{1214} &= m_{12} + m_{13} + m_{14} \\k_{12} &= k_1 + k_2 \\k_{35} &= k_3 + k_4 + k_5 \\k_{68} &= k_6 + k_7 + k_8 \\k_{911} &= k_9 + k_{10} + k_{11} \\k_{1214} &= k_{12} + k_{13} + k_{14}\end{aligned}$$

inertia [M] and stiffness [K] matrices as input data and the string the program results in:

```
>> % pulsations and own frequencies biomechanical hand-forearm-arm model, 8 degrees of freedom
```

```
>> %input data
```

```
>> %mass, [kg]
```

```
>>
```

```
m1=0.014;m2=0.023;m3=0.008;m4=0.012;m5=0.026;m6=0.010;m7=0.015;m8=0.029;m9=0.007;m10=0.010;m11=0.023;m12=0.006;m13=0.007;m14=0.016;m15=0.213;m16=1.0;m17=1.58;
```

```
>> %mass definition
```

```
>> m12=m1+m2;
```

```
>> m35=m3+m4+m5;
```

```
>> m68=m6+m7+m8;
```

```
>> m911=m9+m10+m11
```

```
>> m1214=m12+m13+m14
```

```
>> %stiffness, [N/m]
```

```
>> C=10^3;
```

```
>>
```

```
k1=11.59*C;k2=5.527*C;k3=10.89*C;k4=9.713*C;k5=5.431*C;k6=10.94*C;k7=9.661*C;k8=5.169*C;
```

```
k9=9.146*C;k10=8.187*C;k11=4.804*C;k12=2.994*C;k13=7.761*C;k14=4.675*C;k15=15.43*C;k16=16.33*C;k17= 41.22*C;
```

```
>> % definition of stiffness
```

```
>> k12=k1+k2;
```

```
>> k35=k3+k4+k5;
```

```
>> k68=k6+k7+k8;
```

```
>> k911=k9+k10+k11;
```

```
>> k1214=k12+k13+k14;
```

```
>> %inertia matrix
>> M=[m12 0 0 0 0 0 0;
0 m35 0 0 0 0 0;
0 0 m68 0 0 0 0;
0 0 0 m911 0 0 0;
0 0 0 0 m1214 0 0;
0 0 0 0 0 m15 0;
0 0 0 0 0 0 m16;
0 0 0 0 0 0 0 m17]
```

M =

```
0.0370 0 0 0 0 0 0
0 0.0460 0 0 0 0 0
0 0 0.0540 0 0 0 0
0 0 0 0.0400 0 0 0
0 0 0 0 0.0600 0 0
0 0 0 0 0 0.2130 0
0 0 0 0 0 0 1.0000
0 0 0 0 0 0 0 1.5800
```

```
>> %stiffness matrix
>> K=[k12 0 0 0 0 -2*k12 0 0;
0 k35 0 0 0 -k35/2 0 0;
0 0 k68 0 0 -k68/2 0 0;
0 0 0 k911 0 -k911/2 0 0;
0 0 0 0 k1214 -k1214/2 0 0;
-2*k12 -k35/2 -k68/2 -k911/2 -k1214/2 k15+4*k12+0.25*(k35+k68+k911+k1214) -k15 0;
0 0 0 0 0 -k15 k15+k16 -k16;
0 0 0 0 0 0 -k16 k16+k17]
```

K =

```
1.0e+005 *
0.1712 0 0 0 0 -0.3423 0 0
0 0.2603 0 0 0 -0.1302 0 0
0 0 0.2577 0 0 -0.1288 0 0
0 0 0 0.2214 0 -0.1107 0 0
0 0 0 0 0.2955 -0.1478 0 0
-0.3423 -0.1302 -0.1288 -0.1107 -0.1478 1.0977 -0.1543 0
0 0 0 0 0 -0.1543 0.3176 -0.1633
0 0 0 0 0 0 -0.1633 0.5755
```

```
>> % dynamic matrix calculation D
>> E=inv(M)
```

```
E =
```

```
27.0270 0 0 0 0 0 0 0
0 21.7391 0 0 0 0 0 0
0 0 18.5185 0 0 0 0 0
0 0 0 25.0000 0 0 0 0
0 0 0 0 16.6667 0 0 0
0 0 0 0 0 4.6948 0 0
0 0 0 0 0 0 1.0000 0
0 0 0 0 0 0 0 0.6329
```

```
>> D=E*k
```

```
D =
```

```
1.0e+005 *
4.6262 0 0 0 0 -9.2524 0 0
0 5.6596 0 0 0 -2.8298 0 0
0 0 4.7722 0 0 -2.3861 0 0
0 0 0 5.5343 0 -2.7671 0 0
0 0 0 0 4.9255 -2.4628 0 0
-1.6072 -0.6111 -0.6049 -0.5196 -0.6937 5.1536 -0.7244 0
0 0 0 0 0 -0.1543 0.3176 -0.1633
0 0 0 0 0 0 -0.1034 0.3642
```

```
>> %determination of eigenpulsations - the problem of vectors and eigenvalues
```

```
>> [Q,V]=eig(D);
>> p=sort(sqrt(eig(D))); %own pulsations
>> p1=p(1);
>> p2=p(2);
>> p3=p(3);
>> p4=p(4);
>> p5=p(5);
>> p6=p(6);
>> p7=p(7);
>> p8=p(8);
```

```
>> %determination of own frequencies
```

```
>> f1=p1/(2*pi);
>> f2=p2/(2*pi);
>> f3=p3/(2*pi);
>> f4=p4/(2*pi);
```

```

>> f5=p5/(2*pi);
>> f6=p6/(2*pi);
>> f7=p7/(2*pi);
>> f8=p8/(2*pi);
>> disp(['p1=', num2str(p1), '[rad/s]'])
p1=83.7516[rad/s]
>> disp(['p2=', num2str(p2), '[rad/s]'])
p2=190.6532[rad/s]
>> disp(['p3=', num2str(p3), '[rad/s]'])
p3=245.556[rad/s]
>> disp(['p4=', num2str(p4), '[rad/s]'])
p4=689.7625[rad/s]
>> disp(['p5=', num2str(p5), '[rad/s]'])
p5=699.8519[rad/s]
>> disp(['p6=', num2str(p6), '[rad/s]'])
p6=737.6675[rad/s]
>> disp(['p7=', num2str(p7), '[rad/s]'])
p7=749.1967[rad/s]
>> disp(['p8=', num2str(p8), '[rad/s]'])
p8=980.1244[rad/s]
>> disp('own frequencies:')
natural frequencies:
>> disp(['f1=', num2str(f1), '[Hz]'])
f1=13.3295[Hz]
>> disp(['f2=', num2str(f2), '[Hz]'])
f2=30.3434[Hz]
>> disp(['f3=', num2str(f3), '[Hz]'])
f3=39.0814[Hz]
>> disp(['f4=', num2str(f4), '[Hz]'])
f4=109.7791[Hz]
>> disp(['f5=', num2str(f5), '[Hz]'])
f5=111.3849[Hz]
>> disp(['f6=', num2str(f6), '[Hz]'])
f6=117.4034[Hz]
>> disp(['f7=', num2str(f7), '[Hz]'])
f7=119.2384[Hz]
>> disp(['f8=', num2str(f8), '[Hz]'])
f8=155.9916[Hz]

```

3.2. The elbow joint

The pivoting joint of the elbow joins the humerus bone of the arm with the radius and ulna bones of the forearm. The relationship between the radius and the ulna allows rotary movements of the elbow, the radius bone rotating around the ulna, thus turning the hands.[10]

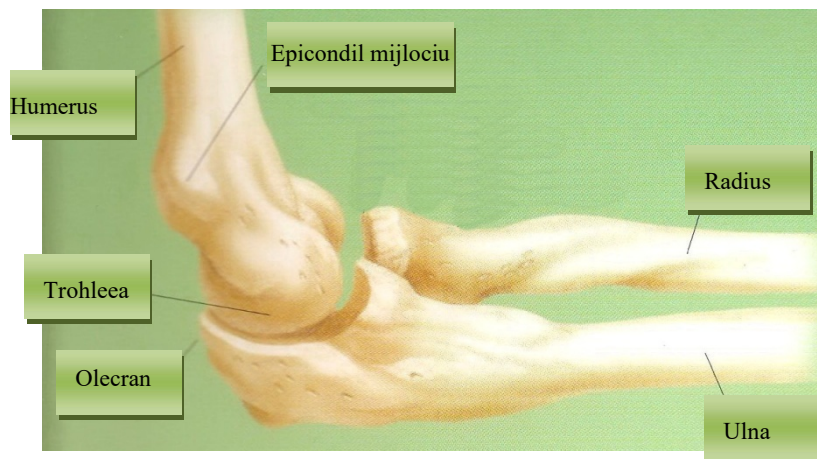


Figure 15 [10]

This joint has only one degree of freedom. It allows the execution of flexion and extension movements of the forearm on the arm. Active flexion and extension have a normal average amplitude of 150° , of which 90° is extension and 60° flexion.

The articular surfaces are:

- *the lower extremity of the humerus* (trochlea, humeral condyle and epicondyle);
- *the upper extremity of the ulna* has a semilunar hollow (great sigmoid cavity), the coronoid apophysis, in the anterior part of the sigmoid cavity and the tuberosity (olecranon), in the posterior part of the sigmoid cavity. At the external edge of this cavity there is another semilunar cavity (small sigmoid cavity) which takes part in the composition of the upper radio-ulnar joint.
- *the upper extremity of the radius* has a cavity that adapts to the humeral condyle.

Articular surfaces they are covered by hyaline cartilage. The humeral trochlea comes in relation to the ulnar notch, and the humeral condyle to the fossa of the radial head.

The means of union are the articular capsule, which connects the humerus with the ulna (ulna) and the radius. The articular capsule is lax and reinforced laterally by *four* stronger ligaments arranged *anteriorly, posterior, external side and internal side*.

The synovial membrane is like a thin sheet that covers the joint capsule from the inside.

The muscles involved in elbow movements are flexors and extensors. *The flexor muscles* are: *biceps brachii* (the most important action is that of supination and, on a secondary level, flexor of the forearm on the arm); *the anterior brachialis* is located under the biceps; *epicondyle muscles* (four in number: brachio-radial, the first external brachialis, the second brachialis, the extensor supinator). *The extensor muscles* are: *triceps brachii* (a voluminous muscle); *anconeus* (a short, triangular muscle, located on the back of the elbow).[6]

Biomechanics of the elbow joint

The elbow joint is a pivot joint and allows only the execution of flexion and extension movements. The flexion movement is the approach of the forearm to the arm. It has a normal active amplitude of nearly 150° . In the final phase of the movement, the hand is not oriented towards the shoulder but towards the chest, because the axis of the forearm does not overlap the axis of the arm, but is directed towards it inside. The explanation lies in the upward and inward oblique orientation of the groove of the humeral trochlea.

The *flexor muscles* are: the anterior brachialis, the biceps brachii and the epicondyle muscles.

The extension movement is the movement of removing the forearm from the arm. The amplitude is 90°. The extension movement is limited by the tip of the olecranon and by the anterior ligament of the elbow which is put under tension.

The *extensor muscles* are: triceps brachii and anconeus (accessory).

By contracting these muscles, the forearm acts as a lever, in which the fulcrum is in the elbow joint.[6]

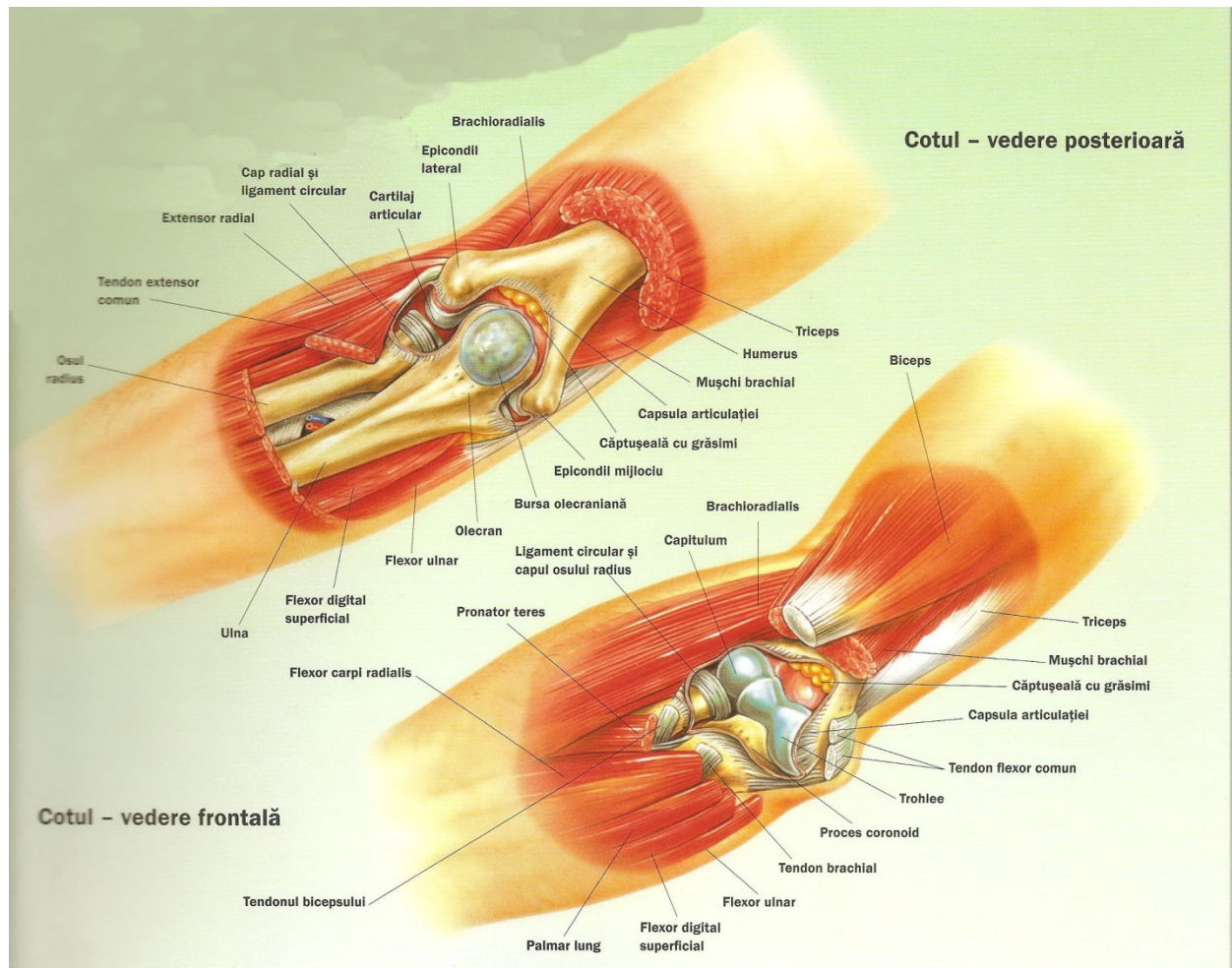


Figure 16 [10]

3.3. Biomechanical models of the elbow joint

A model with two degrees of freedom of the elbow joint, similar to a double pendulum, whose masses evolve in a single vertical plane, with kinematic restrictions imposed on the free end of the second element, is presented in figure 17.

The following assumptions were made for this biomechanical model:

- in the joint acts a resistant moment of elastic and viscous type, both having the linear characteristic in a first approximation;
- the connection between the elements of the model is made by means of cylindrical joints;
- the movement is studied in the vertical plane;
- dry friction forces in the joint were not taken into account.

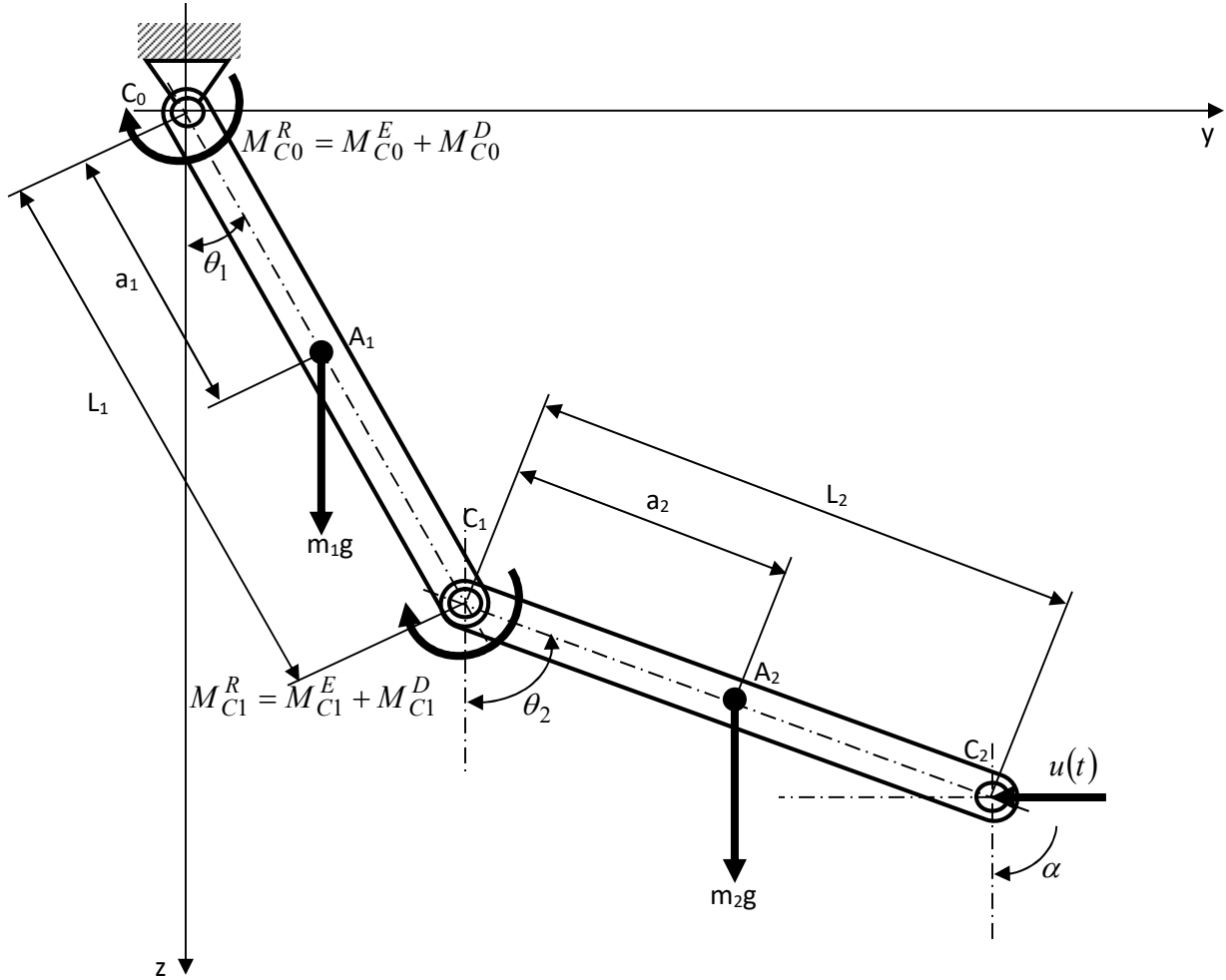


Figure 17

In the shoulder joint C_0 the total resultant moment is:

$$M_{C_0}^R = M_{C_0}^E + M_{C_0}^D$$

where:

$M_{C_0}^E$ - the resultant elastic moment; $M_{C_0}^D$ - the resulting dissipative moment.

The elbow joint C_1 has the total resultant moment:

$$M_{C_1}^R = M_{C_1}^E + M_{C_1}^D$$

with: $M_{C_1}^E$ - the resulting elastic moment; $M_{C_1}^D$ - the resulting dissipative moment.

The system of differential equations of motion according to the model is found with the help of Lagrange's equations of the second kind:

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{q}_i} \right) - \frac{\partial E}{\partial q_i} = \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i}$$

where E is the total kinetic energy of the system, V the potential energy, D the dissipation, and q_i are the generalized coordinates that define the movement of the considered system.

The generalized coordinates are the angles in the two joints (θ_1 and θ_2), between the longitudinal axis of the element and the vertical axis Oz .

The kinetic energy is given by the expression:

$$E = \frac{1}{2} (J_1 \cdot \dot{\theta}_1^2) + \frac{1}{2} (J_2 \cdot \dot{\theta}_2^2) + \frac{1}{2} m_2 v_{A2}^2$$

where J_i are the moments of inertia of each element i evaluated with respect to the articulation point (J_1) and with respect to its own center of gravity (J_2); m_2 is the mass of the forearm; v_{A2} is the velocity of the center of gravity of the forearm.

To evaluate the speed v_{A2} , the coordinates of the center of gravity were determined A_2 :

$$\begin{cases} z_{A2} = L_1 \cos \theta_1 + a_2 \cos \theta_2 \\ y_{A2} = L_1 \sin \theta_1 + a_2 \sin \theta_2 \end{cases}$$

Coordinates of point C_2 , in which the kinematic restrictions are applied, are:

$$\begin{cases} z_{C2} = z_{C20} + u \cos \alpha \\ y_{C2} = y_{C20} + u \sin \alpha \end{cases}$$

and

$$\begin{cases} z_{C20} = L_1 \cos \theta_1 + L_2 \cos \theta_2 \\ y_{C20} = L_1 \sin \theta_1 + L_2 \sin \theta_2 \end{cases}$$

are the initial coordinates of point C_2 and:

$$u = u(t) = u_0 \sin \omega t$$

In the relations above we have the following quantities:

θ_i - the initial values of the two angular coordinates;

L_i - the lengths of the two component elements of the model;

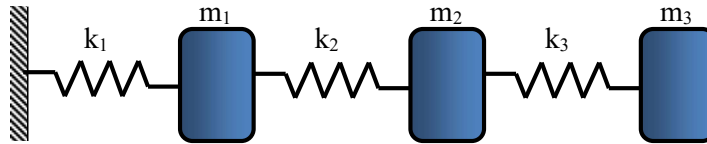
$u(t)$ - the kinematic excitation function of the model;

ω - the angular velocity of the external kinematic disturbance;

u_0 - the amplitude of the external disturbing movement.

Taking into account the expression (4.3), the linear velocity of the mass can be determined m_2 :

$$\begin{cases} \dot{z}_{A2} = -L_1 \dot{\theta}_1 \sin \theta_1 - a_2 \dot{\theta}_2 \sin \theta_2 \\ \dot{y}_{A2} = L_1 \dot{\theta}_1 \cos \theta_1 + a_2 \dot{\theta}_2 \cos \theta_2 \end{cases}$$



To determine the pulsations and natural frequencies for the system with three degrees of freedom hand-forearm-arm, the model of the researchers Suggs and Mishoe (1974) represented in the figure above is used, not taking into account depreciations and using the data from table 1 for calculations.

$$m_1 = m_{17} = 1.58 \text{ kg}$$

$$m_2 = m_{16} = 1 \text{ kg}$$

$$m_3 = m_1 + m_2 + \dots + m_{15} = 0.419 \text{ kg}$$

$$k_1 = k_{17} = 41,22 \cdot 10^3 \text{ N/m}$$

$$k_2 = k_{16} = 16,33 \cdot 10^3 \text{ N/m}$$

$$k_3 = k_1 + k_2 + \dots + k_{15} = 121,918 \cdot 10^3 \text{ N/m}$$

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} 1,58 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0,419 \end{bmatrix} \text{ kg}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 57,55 & -16,33 & 0 \\ -16,33 & 138,248 & -121,918 \\ 0 & -121,918 & 121,918 \end{bmatrix} 10^3 \text{ n/a}$$

```
>> %data entry
```

```
>> m1=1.58; % kg
```

```
>> m2=1; % kg
```

```
>> m3=0.419; % kg
```

```
>> k1=41.22*10^3; %N/m
```

```
>> k2=16.33*10^3; %N/m
```

```
>> k3=121.918*10^3; %N/m
```

```
>> %inertia matrix
```

```
>> M=[m1 0 0;
```

```
0 m2 0;
```

```
0 0 m3]
```

```
M =
```

```
1.5800 0 0
```

```
0 1.0000 0
```

```
0 0 0.4190
```

```
>> % the stiffness matrix
```

```
>> K=[k1+k2 -k2 0;
```

```
-k2 k2+k3 -k3;
```

```
0 -k3 k3]
```

```
K =
```

```
1.0e+005 *
```

```
0.5755 -0.1633 0
```

```
-0.1633 1.3825 -1.2192
0 -1.2192 1.2192
```

```
>> % dynamic matrix calculation D
```

```
>> E=inv(M)
```

```
E =
```

```
0.6329 0 0
```

```
0 1.0000 0
```

```
0 0 2.3866
```

```
>> D=E*K
```

```
D =
```

```
1.0e+005 *
```

```
0.3642 -0.1034 0
```

```
-0.1633 1.3825 -1.2192
```

```
0 -2.9097 2.9097
```

```
>> %determination of eigenpulsations as a problem of vectors and eigenvalues
```

```
>> [Q,V]=eig(D);
```

```
>> p=sort(sqrt(eig(D))); %own pulsations
```

```
>> p1=p(1);
```

```
>> p2=p(2);
```

```
>> p3=p(3);
```

```
>> %own frequencies
```

```
>> f1=p1/(2*pi);
```

```
>> f2=p2/(2*pi);
```

```
>> f3=p3/(2*pi);
```

```
>> %display the results
```

```
>> disp('Display results')
```

```
Display results
```

```
>> disp('Own pulses are:')
```

```
The own pulsations are:
```

```
>> disp(['p1=', num2str(p1), '[rad/s]'])
```

```
p1=85.787[rad/s]
```

```
>> disp(['p2=', num2str(p2), '[rad/s]'])
```

```
p2=200.7436[rad/s]
```

```
>> disp(['p3=', num2str(p3), '[rad/s]'])
```

```
p3=646.5202[rad/s]
```

```
>> disp('Eigenfrequencies are:')
```

```
The natural frequencies are:
```

```
>> disp(['f1=', num2str(f1), '[Hz]'])
```

```
f1=13.6534[Hz]
```

```
>> disp(['f2=', num2str(f2), '[Hz]'])
```

```
f2=31.9493[Hz]
```

```
>> disp(['f3=', num2str(f3), '[Hz]'])
```

```
f3=102.8969[Hz]
```

4. BIOMECHANICAL MODELS OF THE HUMAN ORGANISM

In the following, I will present some models of the human body found in the materials studied for the paper.

In 1977 the researchers Patil, Palanichamy and Ghista made a nonlinear model with seven degrees of freedom (figure 18), complementing the model of Muksian and Nash of 1974 for a human body in the sitting position .

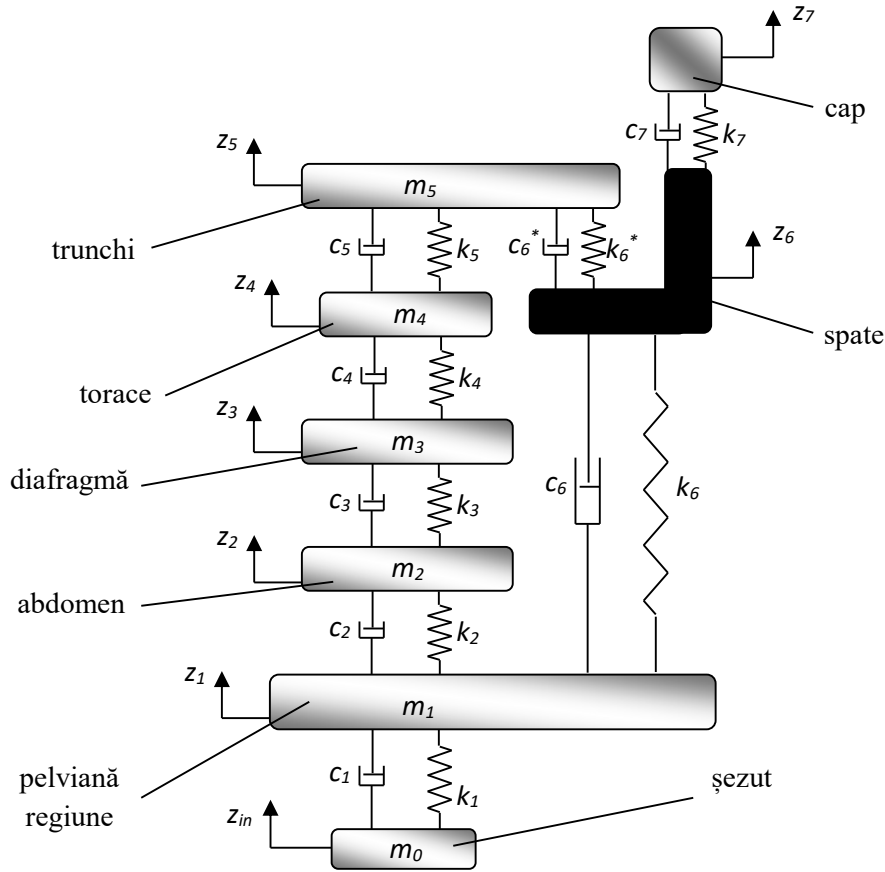


Figure 18 [2]

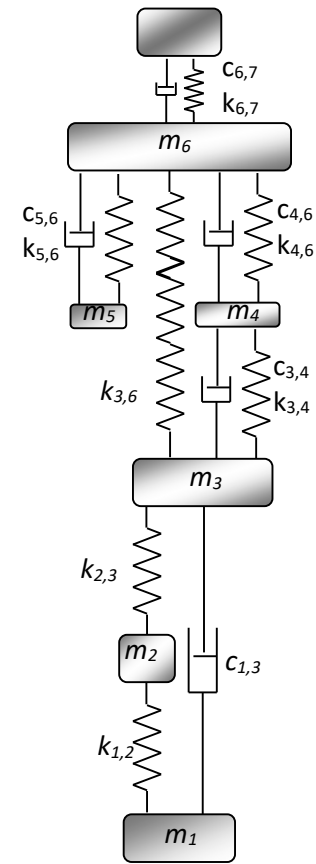


Figure 19 [2]

Professor Magheți presents a model of the human body in a vertical position, in the form of concentrated masses (figure 19).

The International Organization for Standardization published in 1987 the standard ISO 7962:1987 (revised by ISO 5982:2001). It also contained a model with four degrees of freedom of the human body (figure 20) used to determine the transmissibility of head vibrations, in the vertical direction, for the range 0.5 – 31.5 Hz. The standard is limited to sitting and standing positions, and the model does not take into account the non-linearities of the human body.[2]

Also, the model parameters are defined in this standard, with the following values:

[kg]				[N/m]			
m_1	m_2	m_3	m_4	k_1	k_2	k_3	k_4
8.24	8.05	44.85	13.86	$22 \cdot 10^8$	$20.13 \cdot 10^4$	$88.56 \cdot 10^3$	$36,47 \cdot 10^3$

[N/m]				[N*s/m]			
k_1^*	k_2^*	k_3^*	k_4^*	c_1	c_2	c_3	c_4
$36 \cdot 10^7$	$65 \cdot 10^9$	$52.34 \cdot 10^4$	$69.3 \cdot 10^3$	748.1	578	2964	901.8

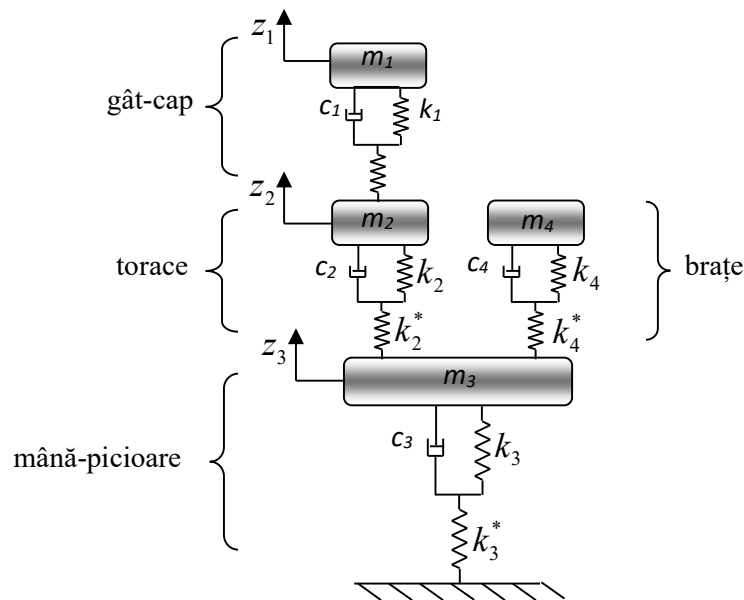


Figure 20

A biomechanical model of the human body under the influence of vibrations, in sitting and standing positions, was created by Cherian in 1990 (figure 21).

Goel and his collaborators presented in 2001 a seven-degree-of-freedom model of a human subject standing on a rigid support (figure 22). The masses were arranged by respecting the anatomical elements, and the values of the damping constants, stiffness coefficients and masses are [2]:

$$m_1 = 6 \text{ kg}, m_2 = 7 \text{ kg}, m_3 = 27,174 \text{ kg},$$

$$m_4 = 32,697 \text{ kg}, m_5 = 6,529 \text{ kg},$$

$$m_6 = 6 \text{ kg}, m_7 = 7 \text{ kg}$$

$$k_1 = k_6 = 310 \text{ n/a}$$

$$k_2 = k_3 = k_4 = k_5 = k_7 = k_8 = 1500 \text{ n/a}$$

$$c_1 = c_6 = 3970 \text{ N} \cdot \text{s/m}$$

$$c_2 = c_3 = c_4 = c_5 = c_7 = c_8 = 100000 \text{ N} \cdot \text{s/m}$$

The masses of the hands are considered to be included in the mass m_4 .

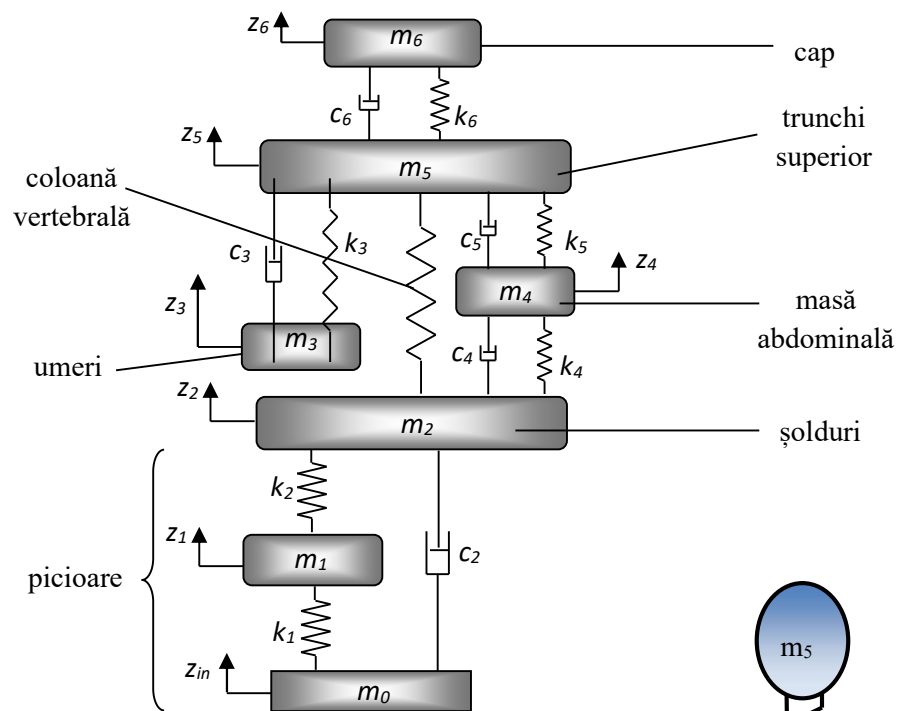


Figure 21

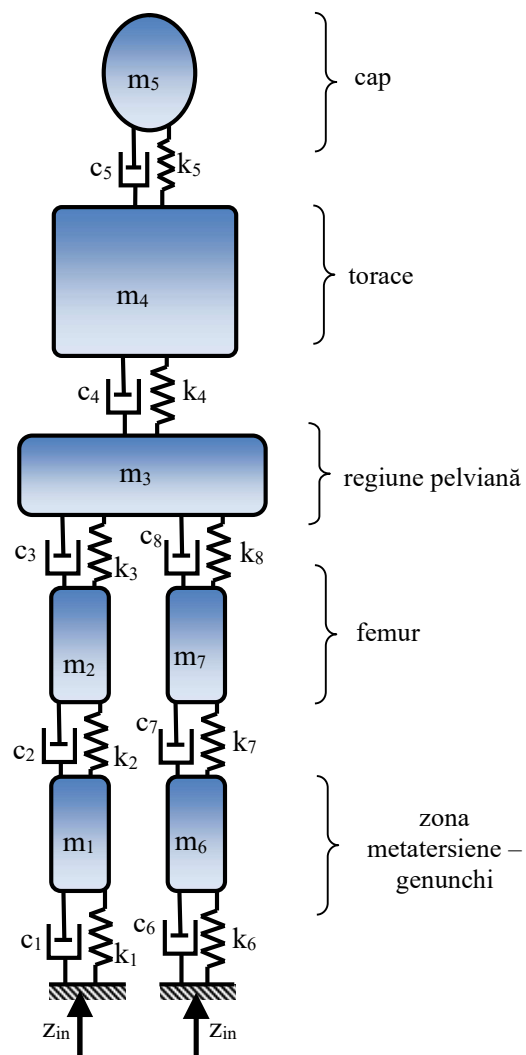
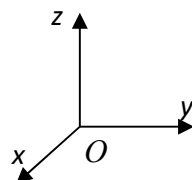


Figure 22

For the model in figure 18, the own frequencies and pulsations will be determined with the help of the Matlab program. The experimental data from the table will be used.

Table 3

	1	2	3	4	5	56	6	7
M [kg]	27.23	5,921	0.455	1,362	32,762	-	6,820	5,450
k [kN/m]	25500	877	877	877	877	52600	52600	52600

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_7 \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_1 + k_2 + k_6 & -k_2 & 0 & 0 & 0 & -k_6 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & 0 & 0 & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 & -k_5 & 0 & 0 \\ 0 & 0 & 0 & -k_5 & k_5 + k_6 & -k_6 & 0 \\ -k_6 & 0 & 0 & 0 & -k_5 & k_5 + k_6 + k_7 & -k_7 \\ 0 & 0 & 0 & 0 & 0 & -k_7 & k_7 \end{bmatrix}$$

% biomechanical model of the human body with 7 degrees of freedom - pulsations and own frequencies

```
>> %data entry
```

```
>> %mass, [kg]
```

```
>> m1=27.23;m2=5.921;m3=0.455;m4=1.3362;m5=32.762;m6=6.820;m7=5.450;
```

```
>> %stiffness, [N/m]
```

```
>> k1=25500;k2=877;k3=877;k4=877;k5=877;k56=52600;k6=52600;k7=52600;
```

```
>> %inertia matrix
```

```
>> M=[m1 0 0 0 0 0 0;
```

```
0 m2 0 0 0 0 0;
```

```
0 0 m3 0 0 0 0;
```

```
0 0 0 m4 0 0 0;
```

```
0 0 0 0 m5 0 0;
```

```
0 0 0 0 0 m6 0;
```

```
0 0 0 0 0 0 m7]
```

M =

```
27.2300 0 0 0 0 0 0
0 5.9210 0 0 0 0 0
0 0 0.4550 0 0 0 0
0 0 0 1.3362 0 0 0
0 0 0 0 32.7620 0 0
0 0 0 0 0 6.8200 0
0 0 0 0 0 0 5.4500
```

```
>> %stiffness matrix
>> K=[k1+k2+k6 -k2 0 0 0 -k6 0;
-k2 k2+k3 -k3 0 0 0 0;
0 -k3 k3+k4 -k4 0 0 0;
0 0 -k4 k4+k5 -k5 0 0;
0 0 0 -k5 k5+k6 -k6 0;
-k6 0 0 0 -k56 k56+k6+k7 -k7;
0 0 0 0 0 -k7 k7]
```

K =

```
78977 -877 0 0 0 -52600 0
-877 1754 -877 0 0 0 0
0 -877 1754 -877 0 0 0
0 0 -877 1754 -877 0 0
0 0 0 -877 53477 -52600 0
-52600 0 0 0 -52600 157800 -52600
0 0 0 0 0 -52600 52600
```

```
>> % dynamic matrix calculation D
>> E=inv(M);
>> D=E*K;
>> %determination of eigenpulsations as a problem of vectors and eigenvalues
>> [Q,V]=eig(D);
>> p=sort(sqrt(eig(D))); %own pulsations
>> %determination of own pulsations
>> p1=p(1);
>> p2=p(2);
>> p3=p(3);
>> p4=p(4);
>> p5=p(5);
>> p6=p(6);
>> p7=p(7);
>> %determination of own frequencies
>> f1=p1/(2*pi);
>> f2=p2/(2*pi);
```

```

>> f3=p3/(2*pi);
>> f4=p4/(2*pi);
>> f5=p5/(2*pi);
>> f6=p6/(2*pi);
>> f7=p7/(2*pi);
>> disp('Results:')
Results:
>> disp('own pulses:')
own pulsations:
>> disp(['p1=', num2str(p1), '[rad/s]'])
p1=12.6954[rad/s]
>> disp(['p2=', num2str(p2), '[rad/s]'])
p2=17.2202[rad/s]
>> disp(['p3=', num2str(p3), '[rad/s]'])
p3=30.7225[rad/s]
>> disp(['p4=', num2str(p4), '[rad/s]'])
p4=48.6351[rad/s]
>> disp(['p5=', num2str(p5), '[rad/s]'])
p5=65.9049[rad/s]
>> disp(['p6=', num2str(p6), '[rad/s]'])
p6=80.4402[rad/s]
>> disp(['p7=', num2str(p7), '[rad/s]'])
p7=167.9425[rad/s]
>> disp('own frequencies:')
natural frequencies:
>> disp(['f1=', num2str(f1), '[Hz]'])
f1=2.0205[Hz]
>> disp(['f2=', num2str(f2), '[Hz]'])
f2=2.7407[Hz]
>> disp(['f3=', num2str(f3), '[Hz]'])
f3=4.8896[Hz]
>> disp(['f4=', num2str(f4), '[Hz]'])
f4=7.7405[Hz]
>> disp(['f5=', num2str(f5), '[Hz]'])
f5=10.4891[Hz]
>> disp(['f6=', num2str(f6), '[Hz]'])
f6=12.8025[Hz]
>> disp(['f7=', num2str(f7), '[Hz]'])
f7=26.7289[Hz]

```

5 . BIOMECHANICAL MODELS OF THE HUMAN KNEE JOINT

5.1. The knee

The knee is one of the most complex joints in the body. It connects the thigh and calf.

The knee skeleton consists of the lower extremity of the femur (thigh bone), the upper extremities of the two bones of the calf: the tibia and the fibula, and the region's own bone, the patella (figure 18).

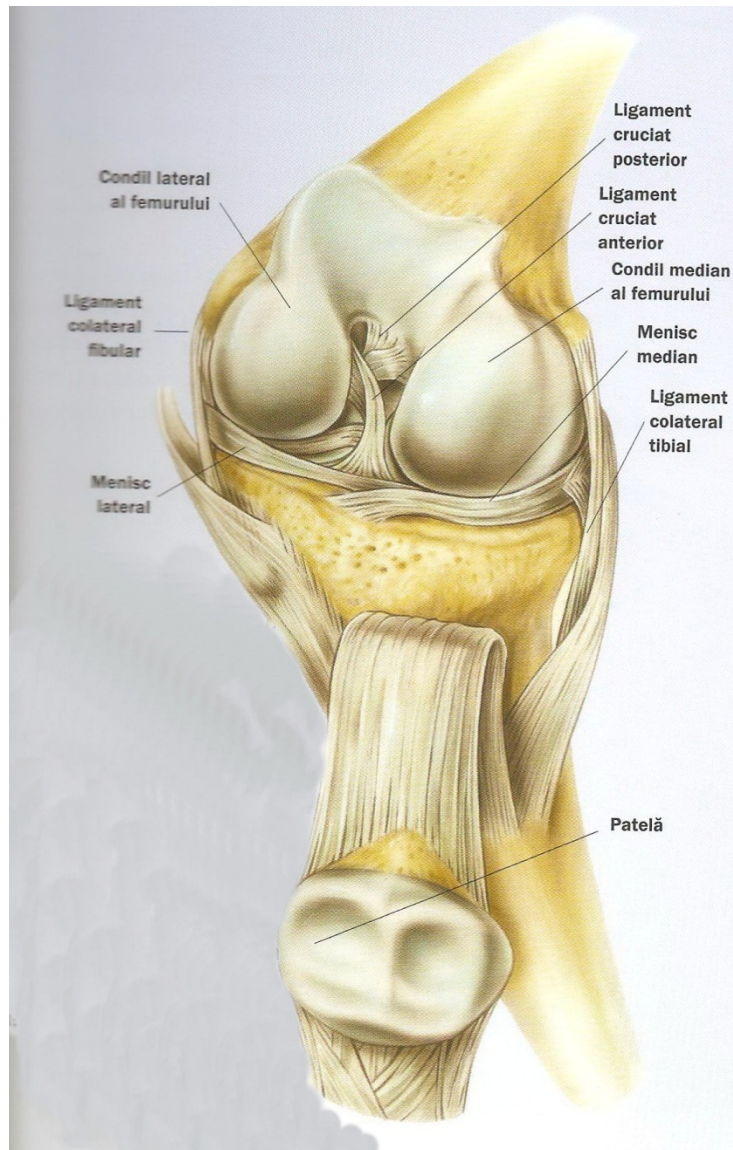


Figure 18 [10]

This joint, like the elbow, belongs to the category of pivoting joints (which accepts movement in one plane, example: bending/straightening the knee) (figure 19). It does, however,

allow some forward and backward gliding movements and a reduced level of rotational movements. The knee is a joint capable of lifting weights.



Figure 19 [10]

The three bones that meet at the knee and the joint cavity between them are united by the capsule of the knee joint. This capsule has a membrane on the edge and is filled with synovial fluid that lubricates the surfaces of the joint. The base of the femur and the head of the tibia have a cartilaginous layer that supports the slow movement of the knee. The patella is located in the tendon of the quadriceps muscle and has an approximately triangular shape. The anterior face is convex and comes into contact with the fascia of the knee and with the integuments. The back face is concave. On its base and edges, the quadriceps tendon is inserted, and at the top, the patellar tendon.[10],[6]

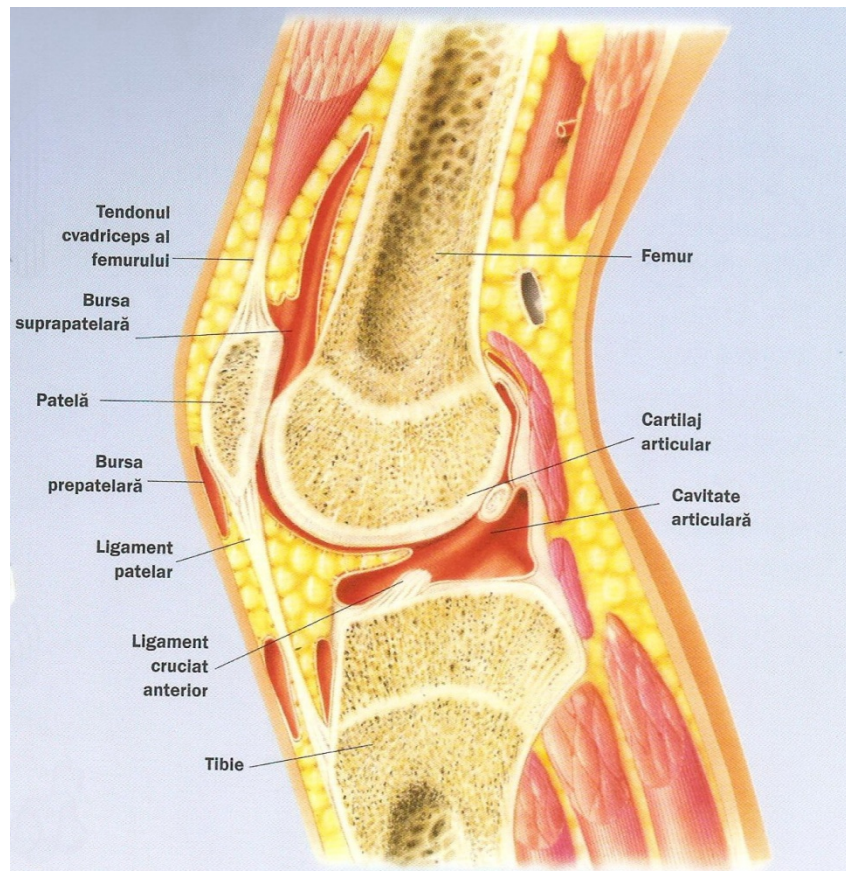


Figure 20

Since the bones of the knee joint cannot hold themselves in position, a network of strong ligaments and muscles is necessary to provide strength and stability. The patella is held in position by strong ligaments that accept the longitudinal sliding movement of the patella above the femur. The collateral ligaments are arranged along the outer sides of the joint capsule, while inside the capsule, the cruciate ligaments play a role in controlling the movement of the tibia and patella. Of equal importance in maintaining stability are the muscles that support the knee.[10]

At the knee level, there are three joints: femur-tibial (the actual knee joint), femur-patella (participating in the composition of the knee joint) and the superior tibio-fibular joint.

The femoral-tibial joint is the largest and strongest joint of the body. Structurally, it is an imperfect trochlearthrosis and therefore has two menisci in its constitution. The lower extremity of the femur has the two condyles, separated from the intercondylar cavity and the trochlea and covered on the surface by a hyaline cartilage. The upper extremity of the tibia presents two glenoid cavities covered by hyaline cartilage, separated from each other by two tubercles (internal and external) of the bone massif belonging to the tibial spine. The distal ends of the cruciate ligaments are inserted on the tibial spine. The posterior face of the patella is divided into two lateral facets by a chamfered ridge and is covered by hyaline cartilage.

Since there is no perfect congruence between the articular bone surfaces of the femur and tibia, between them it developed, on each cavity glenoid one meniscus each. The external meniscus has a circular shape, and the internal one the shape of the letter C.[6]

Internal meniscus, through its anterior horn, it is fixed to the anterior edge of the tibial plateau, immediately in front of the anterior cruciate ligament, and through its posterior horn, on the retrosplinal surface, immediately behind the insertion of the posterior cruciate ligament.

The external meniscus, through its anterior horn, is fixed on the presplinal surface, immediately before the spine and on the external face of the anterior cruciate ligament, and through its posterior horn, it is fixed on the internal tubercle of the tibial spine. The two menisci are joined at their anterior part by a delicate formation called the transverse ligament which is surrounded by the anterior fat cell bundle of the knee.

These menisci, not being strictly cartilaginous, have greater elasticity and deformability than normal cartilage. The internal part of the meniscus does not contain vessels, but in the capsular part they are abundant.

The bone segments in the joint are held together by a joint capsule strengthened by six ligaments. The articular capsule is a fibrous sleeve, which is fixed all around, very close to the border of the articular cartilages, laterally on the menisci and forward on the jugal ligament, reaching the tibia. It is very resistant, it can withstand pulls greater than 300 kg. The six ligaments are:

1. the anterior (patellar) *ligament* – represents the terminal tendon of the quadriceps, it stretches from the patella to the anterior tuberosity of the tibia, it is widened transversely, thick and very resistant.
2. *the posterior ligament* (Winslov) – it is confused with the insertions of the twin muscles (of the triceps surae). The middle part is in the intercondylar cavity and is confused with the insertions of the ligaments crossed.
3. *the internal lateral ligament* - they are inserted high on the tuberosity of the internal femoral condyle, and low, on the uppermost part of the internal face of the tibia.
4. *external lateral ligament* - they are inserted high on the tuberosity of the external femoral condyle, and low, on the antero-external side of the head of the fibula.

Cruciate ligaments are found in the intercondylar cavity.

5. *the anterior cruciate ligament* inserts above, on the posterior portion of the external condyle and goes down, forward and inward to insert on the antero-internal part of the tibial spine and on the prespinal rough surface, between the insertions of the anterior horns of the menisci.

6. *the posterior cruciate ligament* inserts on the posterior portion of the internal condyle and runs downward, forward, and inward to insert behind the tibial spine.

7. *the synovium of the knee* covers the inner face of the capsule; it adapts to all the bottoms of the capsular bag and is interrupted at the level of the insertion of the menisci, dividing into two portions: one suprameniscal, which represents almost the entire synovium, and another submeniscal, much smaller in size. The synovium of the knee communicates in almost 1% of cases with the synovium of the superior tibio-fibular joint. [6]

The femoral-patellar joint is a trochlearthrosis, being made up of the trochlea of the lower extremity of the femur and the posterior face articular of the patella. The capsulo-ligamentary apparatus is confused with that of anterior face of the femur-tibial joint.

The muscles involved in knee movements are:

a) *thigh muscles* (anterior: quadriceps, tensor fascia lata, internal rectus, tailor and posterior: hamstrings), biarticular muscles;

b) *calf muscles* , popliteus and thin plantar.

Static knee

The biomechanical axis of the femur which, passing through the center of the femoral head and through the intercondylar cavity, makes a 10° angle open upwards with the anatomical axis of the femoral body. Relative to the anatomical axis of the tibia, the anatomical axis of the femur is slightly inclined outwards, thus forming an open angle of 170° - 177°.

Biomechanics of the femur-tibial joint

The femoral-tibial joint has only one degree of freedom and consequently presents two main movements: flexion and extension of the calf on the thigh, movements to which are added secondary ones such as: internal rotation and external rotation. The joint also shows very low amplitude lateral tilting movements. The average amplitude of active flexion and extension movements is 135°, and of passive ones 150°. The movements are performed in the sagittal plane, around a transverse axis that passes through the two femoral condyles. The femur-tibial joint works according to the principle of a third degree lever, by moving the femur on the fixed tibia (example: support on the ground), by moving the tibia on the fixed femur (example: sitting position) or by moving the two bones simultaneously (example: when walking, when the leg is pendulous).

The flexion movement is the one through which the back of the calf approaches the back of the thigh. It runs around several axes. The beginning of the flexion movement is done more by rolling, and the end more by rotating in place around a fixed axis. When the flexion reaches 70°, an internal rotation movement is also associated, which can reach up to 20° amplitude. The motor muscles for flexion are: the biceps femoris and the semimembranosus, as the main muscles, but the semitendinosus, the twins, the popliteus, the thin plantaris, the internal rectus and the tailor also intervene. The limitation of the flexion movement is achieved by the meeting of the posterior face of the calf with the posterior face of the thigh.[6]

The extension movement is the one through which the back of the calf moves away from the back of the thigh. At the beginning of the movement, the extremity of the femur rotates, then it rolls on the tibial plateau, until the long axis of the calf ends up continuing the long axis of the

thigh. The extension movement is also associated with an outward rotation movement of the calf on the thigh. Motor muscles of extension they are primarily the quadriceps and the tensor fascia lata. Together with the quadriceps tendon, the patella, the patellar fins and the patellar tendon, they form a complex knee extension apparatus. The extensors act with all their force when forcefully extending the flexed knee or when forcefully locking the knee in slight flexion, as in physical activity. The extension movement is limited by the posterior ligament of the joint, the anterior cruciate ligament, and accessory by the posterior cruciate ligament, the hamstring muscles and the anterior ligaments that extend during extension.

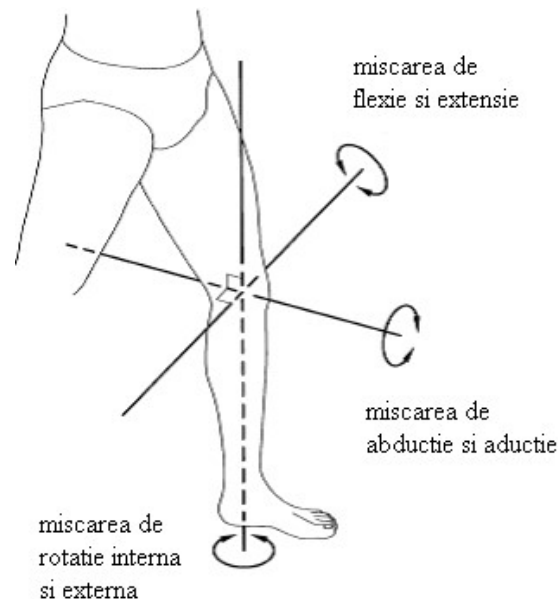


Figure 21 [Vaughan, 1999]

Internal and external rotation movements are associated with flexion and extension movements. The cruciate ligaments also intervene, which rotate the lower leg outwards the final position of flexion and into the final position of extension. The amplitude of the active rotation movement is 15° - 20° , and of passive rotation of 35° - 40° . External rotation is performed by the biceps femoris and rotation. Inside it is performed by: semimembranosus, semitendinosus, popliteus, rectus intern and tailor. In external rotation, the lateral ligaments expand again the cruciate ligaments relax, in time in the internal rotation se the cruciate ligaments are stretched and the lateral ligaments are stretched.

Lateral movements are limited by the lateral ligaments especially during walking, when they are put under maximum tension with knee extension. In full flexion, the external lateral ligament relaxes, but the internal one remains slightly stretched. In semiflexion, however, maximum relaxation of the ligaments is obtained. Cruciate ligaments limit forward and backward movement of the tibial plateau on the femoral condyles, when the knee is extended. The anterior cruciate ligament limits forward movement, and the posterior one - backward movement. The anterior cruciate ligament stretches in extension, relaxes in slight flexion, and stretches again in hyperextension. The posterior cruciate ligament stretches in full flexion, relaxes in semiflexion, and stretches again slightly in extension.[6]

Biomechanics of the menisci

Although solitary on the tibia, the menisci move in flexion, from front to back on the tibial plateau, but they also approach each other slightly, through the posterior extremities. In extension, the menisci move in the opposite direction, i.e. back to front, touch the front edges of the tibial plateau and move slightly away from each other. They also move together with the tibial plateau in relation to the femoral condyles, always being on that part of the plateau that bears the pressure of the condyles. Thus, in extension, the condyles slide forward, pushing the menisci in front of them, and in flexion, the condyles slide backward, pushing the menisci behind them. In external rotation of the calf, the anterior part of the medial meniscus follows the capsule to which it attaches and moves posteriorly forward and internally outward, while its posterior part is pushed back by the femoral condyle.

The role of the menisci:

1. It fills the free space between the curved surface of the femur and the flat surface of the tibia and thus prevents the protrusion of the synovium and the capsule into the joint cavity during movements.
2. Centers the support of the femur on the tibia during movements.
3. It participates in the lubrication of the joint surfaces, ensuring the uniform distribution of the synovium on the surface of the cartilages.
4. Plays the role of a shock absorber between the bone extremities, especially in hyperextension and hyperflexion movements.
5. They significantly reduce the friction between the bone extremities.[6]

Biomechanics of the femoral-patellar joint

The patella is held in place by a complicated system of reins, of muscular, ligamentous and tendinous origin. In the vertical direction, it is fixed by the patellar tendon and the quadriceps tendon - only it is a motor - and stresses the patella, pulling it out and applying it strongly in the trochlear groove. These tendons make an outward open angle between them. In the transverse direction, the patella is supported by the two patellar fins. The internal fin extends from the internal edge of the patella, to the internal face of the internal condyle, it is strengthened by the insertion of the internal vastus and the internal meniscus-patellar ligament and is particularly stressed. The external fin extends from the external edge of the patella, to the external face of the external condyle, is strengthened by the vastus externalus, fascia lata and the external meniscus-patellar ligament and is less developed. Outside of these formations, a series of fibrous elements cross over the patella, forming a veritable network. It is about the expansions of the tailor, the fascia lata, the aponeurosis gambiere and the rectus anterior.

The role of the patella:

- in extension, keeps the tendon away from the femoral trochlea;
- increases the lever arm of the quadriceps, displacing the quadriceps tendon relative to the rotation axis of the knee, easing the activity of this muscle;
- in flexion, being pulled by the patellar tendon, the patella progressively contacts the articular surface of the trochlea and enters the trochlear groove; starting from the top and slightly from the outside, it descends towards the median line, passes over the vertical line of the trochlea, then, once it enters the groove between the two condyles, it goes outwards again, so that at the end of the flexion movement it covers the condyle exclusively external.[6]

Integrated into the triple extension chain of the lower limb, the knee joint actively participates in daily activities. To carry out daily activities, the knee uses approximately

110° flexion-extension and 10–15° abduction-adduction. Normal walking involves from full extension to 70° flexion, and going down the stairs to 90° flexion. The sitting position in an armchair requires more than 90°.

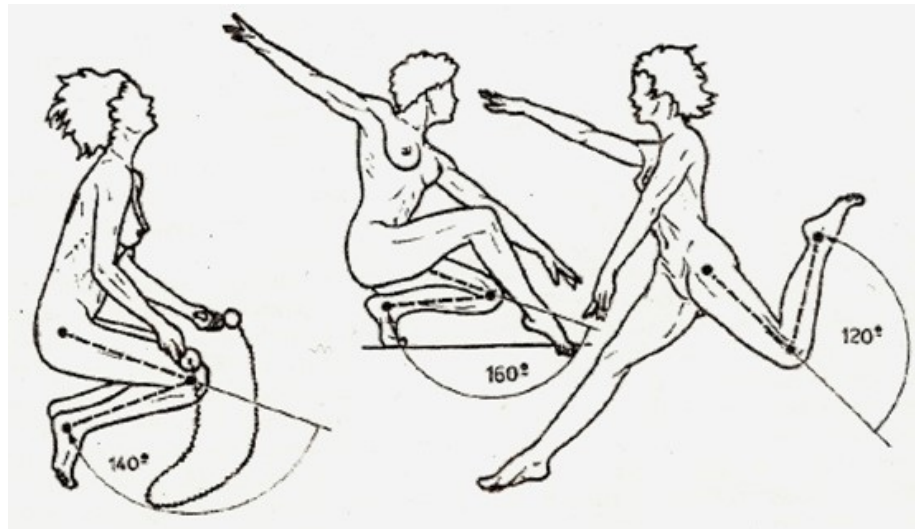


Figure 22 [Popescu and Trandafir, 1998]

Regarding the movement of the articular surfaces, it should be specified that in the femoral-tibial joint, although the movement is carried out in all planes simultaneously, it is minimal in the transverse and frontal planes, and in the femoral-patellar joint, the movement is carried out in the frontal and transverse planes simultaneously.

The axes of the femoral and tibial diaphyses form an open lateral angle of 173°. On the other hand, the rotation centers of the three main joints of the lower limb are located in a frontal plane on the same line called the mechanical axis of the lower limb. The mechanical axis overlaps and coincides with the anatomical one only at the level of the calf. At thigh level, the anatomical axis is laterally offset 7° from the mechanical one. Due to the inter-articular distance of the hips greater than the distance between the ankles (the support polygon), the mechanical axis is at its most oblique downwards and medially 3° facing the vertical, and perhaps even more oblique, the wider the pelvis is.

The joint is in balance and the projection of the body's center of gravity falls inside the support polygon. In the *bilateral support position*, the knees support 85.6% of the body weight. In the *unilateral support position*, the joint supports the weight of the supraarticular body and the pendulous lower limb, i.e. 93% of the total body weight. [4]

Geometry of articular surfaces.

The surfaces of the proximal tibia and the distal femur have specific shapes that allow one to exceed the other. The femoral condyles have a variable profile (figure 23, table 4). The diameters of the tibial plateaus are larger than the diameters of the corresponding femoral condyles (figure 24, table 5).

The dimensions of the tibial plateau are smaller than the distances of the femoral condyles. The medial condyle of the tibia is concave superior (the center of curvature is located

above the tibial surface) with a radius of curvature of 80 mm (Kapandji, 1987). The lateral condyle is superior convex (the center of curvature is located below the tibial surface) with a radius of curvature of 70 mm (Kapandji, 1987). The shape of the femoral surfaces is complementary to the shape of the tibial plateau. The shape of the posterior femoral condyles can be approximated by spherical surfaces.

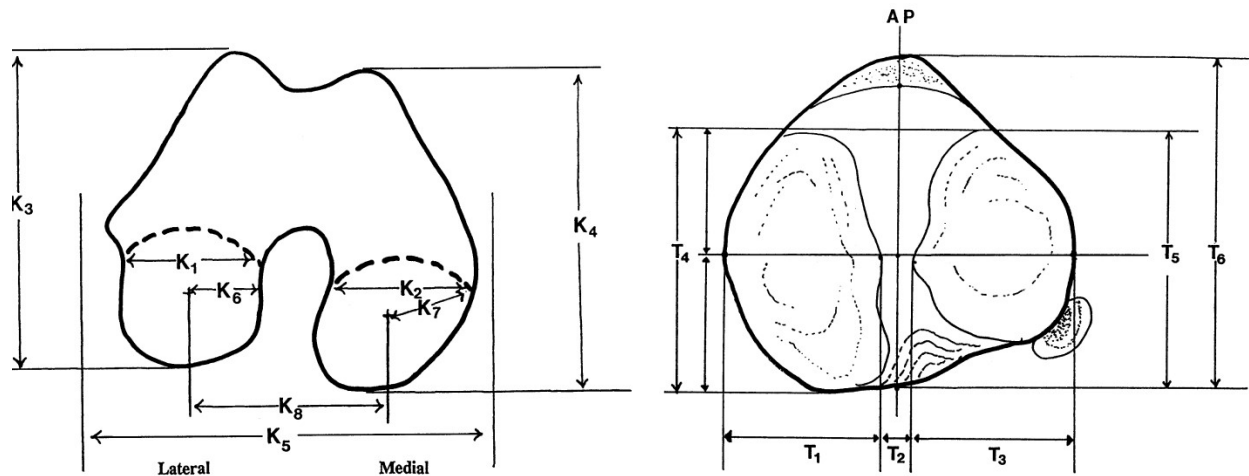


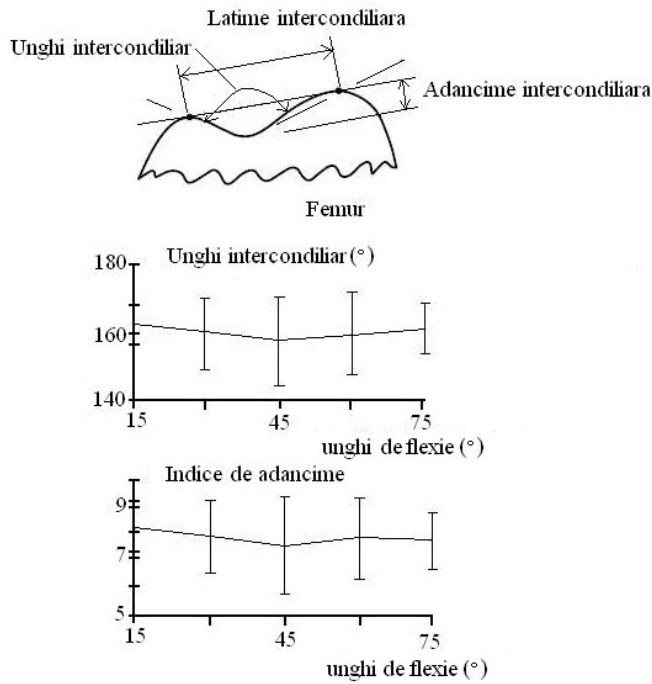
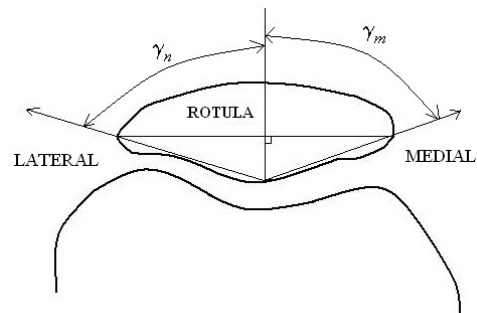
Figure 23 (Kaufman and Kai-Nan, 2000) **Figure 24** (Kaufman and Kai-Nan, 2000)

Table 4

		CONDITIONS				
		Side		Medial		
Parameters	Symbo l	Distance (mm)	Symbo l	Distance (mm)	Symbo l	Distance (mm)
Distance medial/lateral	K1	31 ± 2.3 (male) 28 ± 1.8 (female)	K2	32 ± 3.1 (male) 27 ± 3.1 (female)		
Distance anterior/posterior	K3	72 ± 4.0 (male) 65 ± 3.7 (female)	K4	70 ± 4.3 (male) 63 ± 4.5 (female)		
The radii of curvature of sup. posterior condyle	K6	19.2 ± 1.7	K7	20.8 ± 2.4		
Epicondylar width					K5	90 ± 6 (male) 80 ± 6 (female)
The distance between the sup centers. condylar					K8	45.9 ± 3.4

Table 5

Parameters	symbols	General	Male	Female
Tibial plateau widths (mm)				
The medial plateau	T1	32 ± 3.8	34 ± 3.9	30 ± 2.2
The side plate	T3	33 ± 2.6	35 ± 1.9	31 ± 1.7
Total width	T1+T2+T3	76 ± 6.2	81 ± 4.5	73 ± 4.5
Tibial plateau lengths (mm)				
Medial length	Q4	48 ± 5.0	52 ± 3.4	45 ± 4.1
Side length	T5	42 ± 3.7	45 ± 3.1	40 ± 2.3
Interspinal distance(mm)	T2	12 ± 1.7	12 ± 0.9	12 ± 2.2
Length of the intercondylar area (mm)	T6	48 ± 5.9	52 ± 5.7	45 ± 3.9

**Figure 25****Figure 26**

The geometry of the patello-femoral joint surfaces remains relatively constant during knee flexion. The intercondylar angle, the angle formed by the half lines that pass through the minimum and maximum points of the condylar profile, changes only with $\pm 3,4^\circ$ for a knee flexion from 15 to 75° (figure 25). Lateral and medial patellar angles change by less than one degree during full knee flexion (figure 26). [4]

5.2. Biomechanical models of the knee

The movement of the knee takes place in a wider framework of the global configuration of the movement of the entire human body, when it performs a certain activity. Its movement is the result of the complex interaction of all parts of the body that take part in the performance of that activity, be it walking, running or other movements, so mathematical modeling must take into account the physical, geometric and dynamic constraints that exist in the knee at every moment imposed by the movement of the whole body.

The joint can be mathematically modeled using a transformation matrix (Nordin and Frankel, 1989). Parametrizing the transformation we can manipulate the degrees of freedom of the joint movement between the four bones of the knee joint, so that we can describe the movement.[3]

In the dynamics of the knee joint, the shape of the bones and the position of the ligaments represent an important problem. Usually, the tibial plateau and the condyles are modeled by geometric approximations or by descriptions using the finite element method, based on computed tomography or magnetic resonance imaging (Frey, Riener, Michas, Regenfelder, Burgkart, 2006). Regarding the modeling of the ligaments, we recall the work of the authors Zheng, Fleisig, Escamilla, Barrentine (1998), in which the ligaments are modeled as spring-damper type elements. The authors propose an analytical model in the sagittal plane for ligaments, muscles and for the contact forces between bones and menisci, with the aim of examining joint forces during movement.

Many works model the ligamentous structures and the posterior capsular tissue of the knee joint through non-linear elastic elements, which take the external loads only when they are in tension. Other works model the passive movement of the joint, without loading, using as constraint elements three ligaments and the lateral and medial compartments of the knee (Wilson, Feikes and O'Conner, 1998; Feikes, O'Conner and Zavatsky, 2000). In this case, surface modeling errors can generate errors in kinematic and kinetic simulations of joint contacts.

The kinematic analysis of the movements of the knee joint is generally done from two perspectives: a) *the actual movements of the limb components* interconnected in the joints and b) *detailed analysis of the movements of the joint surfaces* (Kaufman and Kai-Nan An, 2000). The effective movements are 3D joint rotations described with the help of Euler's angles. The 3D free translations and rotations of the joints can also be described with the help of the screw axis system (Kinzel, Hall and Hillberry, 1972; Spoor and Veldpaus, 1980; Woltring, Huiskes, deLange and Veldpaus, 1985).

The classification of the biomechanical models of the human knee can also be made according to the planes in which the movement is performed: a) *2D models* that consider the movement only in the sagittal plane and b) *3D models* (Shelburne. Pandy, Anderson, Torry, 2004). The movement of the surfaces in the femoral-tibial joint is minimal in the transverse and frontal planes, although it is carried out in all planes simultaneously. In the femoral-patellar joint, the movement is carried out in the frontal and transverse planes simultaneously.[4]

The 2D models described by Morrison (1970), Moeinzadeh (1981), Moeinzadeh, Engin and Akkas (1983), Moeinzadeh and Engin (1983), (1988), Tumer and Engin (1993), Tumer, Wang and Akkas (1995), Abdel-Raham and Hefzy (1991), (1993) and Ling, Guo and Boersma (1997) can be understood as different versions of the same dynamic model consisting of two rigid bodies, the fixed femur and the mobile tibia connected by ligaments and having a single point of contact.

Hefzy and Abdel-Raham (2000) describe a 3D version that adopts the same coordinate system introduced by Grood and Suntay (1983).

Starting with Elftman (1939), research attempts to model the movement of joints by exchanging different input data with output data; the input data, such as the forces and moments applying to the joints, being determined from experimental measurements regarding the movement. The mentioned models cover the following research areas: a) joint structure, b) movement mechanism formation, c) kinematic analysis, d) kinetostatic analysis with force classification, e) dynamics synthesis, laws of motion, geometric parameters, bone profile modeling.

In the work of the authors Moeinzadeh and Engin (1988) the movement is described by reducing a second-order nonlinear system of ordinary differential equations to a linearized algebraic system. Thus, it is shown that the extension of the knee causes the elongation of the lateral ligaments and the anterior cruciate ligament, which supports most of the load, but not the posterior cruciate ligament.

Research groups Lew and Lewis (1978), Loch, Lou, Lewis and Stewart (1992) presented empirically determined 3D models for ligament forces and knee joint contact. Knee joint stiffness matrices were compared with experimental data, the model being used in a predictive manner. Similar studies were carried out by Grood and Hefzy (1986) and Wismans, Veldpaus, Janssen, Huson and Struben (1980).

The number of theoretical works is small compared to the experimental and practical (surgical) works. The studied area is restricted and specialized for a certain purpose, breaking the joint from the rest of the human body, which considerably influences the movement of each joint. The movement of the knee joint depends on the movement of the entire human body.

For modeling the movement of the knee joint, the shape of the bones and the position of the ligaments matter. Usually, the tibial plateau and condyles are modeled by geometric approximations or by finite element descriptions, obtained from computed tomography or magnetic resonance imaging (Frey, 2006). The finite element method (FEM) is used both in the study of joint movement simulation problems and for the express determination of contact stresses in a mobile joint or for the determination of friction in an artificial joint. When modeling the articular surfaces using the MEF method, details and unevenness of the surfaces are often neglected, because the geometry of the surfaces cannot be described with the help of a single function. Other works model the passive motion of the joint without loading using five constraint elements: three ligaments and the lateral and medial compartments of the knee (Wilson, Feikes, & O'Conner, 1998; Feikes, O'Conner, & Zavatsky, 2000). Thus, surface errors can generate important errors in kinematic and kinetic simulations of joint contacts.[4]

In the following, some of the relationships used to describe the surfaces of the femur and tibia are presented. In the work of the authors Hefzy and Abdel-Raham (2000) the femoral articular surfaces are studied as spherical parts while the tibial plateaus are considered as flat surfaces. The equation of the femoral spheres, lateral and medial, described in the coordinate system attached to the femur is:

$$f(x, y) = -\sqrt{r^2 - (x - h)^2 - (y - k)^2} + l$$

in which the values of the parameters r , h , k , and l are 21; 23.75; 18; 12 mm and 20; 2.3; 16; 11.5 mm for the lateral and medial spheres respectively. The equation of the lateral and medial tibial planes described in the coordinate system attached to the tibia is

$$g(x', y') = my' = c$$

where the values of the parameters m , c are 0.358; 213 mm and -0.341 ; 212.9 mm for the lateral and medial planes respectively. A three-dimensional model of the knee joint representing the articular surfaces (cruciate ligaments, 1-4; collateral ligaments, 5-8 and capsule structure, 9-12) is shown in figure 27.

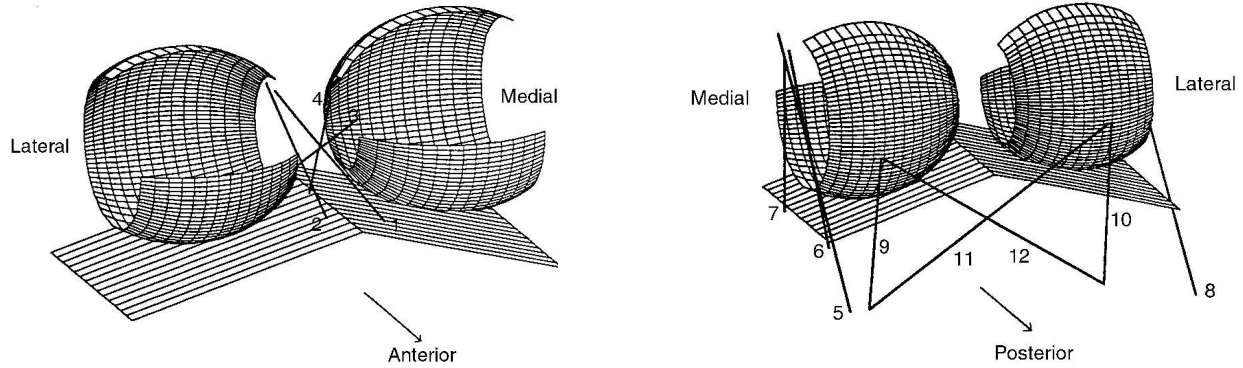


Figure 27 [Hefzy and Abdel-Raham, 2000]

The modeling of the articular surface of the knee must verify the conditions of natural and geometric compatibility. The geometric compatibility conditions assume that the distance between the contact points of the two surfaces is zero, so that penetration is avoided. The natural compatibility conditions ensure that the normals to the two surfaces at the contact points are collinear.

Shunji Hirokawa, Takashi Ueki, Ayaka Ohtsuki (2004) model knee joint surfaces with irregularities also using a parametric polynomial function and Fourier series.

In the works of Wismans, Veldpaus, Janssen, Huson and Struben (1980), Engin and Moeinzadeh (1982), Blankvoort, Kuiper, Huiskes and Grootenboer (1991), Tumer and Engin (1993), the geometry of the surfaces of the two bones is described with the help of two 2D polynomials. In general, it can be observed that small errors can lead to penetrations of the two bones in contact. Delp and Loan (1995) present a method that uses polygons to represent bone surfaces. Here, the natural compatibility conditions are not necessarily satisfied, so the normals at the contact points are not collinear. In the works of Hart (1974), Scherrer (1977), Scherrer and Hillberry (1979), Ateshian, Soslowky and Mow (1991), Hirokawa (1991), Almond (1991), Hefzy and Yang (1993), parametric surfaces with using cubic spline functions.

Mathematical formulations such as Bezier curves (Pierre Bezier 1962), B-spline and NURBS (Farin 1992) are standardized tools for creating free forms, but they are not close enough for modeling articular surfaces. [V. Moşneguţu, PhD Thesis, 2008]

In order to represent the various ligamentous structures and the posterior capsular tissue of the knee joint, nonlinear elastic elements are used in many works that take the load only when they are in tension.

The deformation of the ligament elements is given by Hefzy and Abdel-Raham (2000), by the relation below and the threshold value is specified for $\varepsilon_1 = 0.03$.

$$\varepsilon_j = \frac{L_j - L_{0j}}{L_{0j}}$$

where L_j, L_{0j} are the current and initial lengths of the ligaments. Also using

$$\varepsilon_j = \frac{L_j|_{\text{in extensio completa}}}{L_{0j}} \text{ can calculate the initial length of each ligament.}$$

For the size of the ligament forces, the authors Wismans, Veldpaus, Janssen, Huson and Struben (1980), Blankevoort et al. (1991), Woo, Johnson and Smith (1993), Tumer and Engin (1993) used the force-elongation relationship. For the j ligamentous elements it is proposed

$$F_j = \begin{cases} 0 & ; \varepsilon_j \leq 0 \\ K1_j(L_j - L_{0j})^2 & ; 0 \leq \varepsilon_j \leq 2\varepsilon_1 \\ K2_j(L_j - (1 + \varepsilon_1)L_{0j})^2 & ; \varepsilon_j \geq 2\varepsilon_1 \end{cases}$$

where $K1_j$ and $K2_j$ are the stiffness coefficients of the j elastic elements, for the linear and parabolic zones respectively, and L_j and L_{0j} are the initial and current lengths respectively.

In the paper Zhi-Kui Ling, Hu-Qing Guo, Stancey Boersma (1997) the following relation expresses the non-linearity of the ligament forces for the lateral and cruciate ligaments,

$$f_{mag_j} = \begin{cases} k_j(\text{inow}_j - \text{lstart}_j)^2; & \text{daca } \text{inow}_j \geq \text{lstart}_j \\ 0 & ; \text{daca } \text{inow}_j \leq \text{lstart}_j \end{cases}$$

where j represents the index of the different ligaments, inow the current length of the ligament and lstart the length of the stretched ligament, which is obtained from the initial length (the length of the ligament in maximum extension) multiplied by its own deformation index ε_j ,

k_j represents the stiffness, and its volumes correspond to the data from the literature.

$$\text{lstart}_j = \text{initial}_j \times \varepsilon_j$$

where the deformation index ε_j is also taken from the literature.

To calculate the length of each ligament, the authors Chittajallu and Kohrt (1995) use the distance formula:

$$L_i = \sqrt{(X_{Ti} - Y_{Fi})^2 - (Y_{Ti} - Y_{Fi})^2}$$

where X_{Ti}, Y_{Ti} are the coordinates of the insertion point on the tibia of the ligaments in the tibial coordinate system. Here the resultant force in each ligament due to elongation/contraction, F_i , is given by

$$F_i = F_{is} \cdot \frac{\Delta L_i}{L_{i0}} = F_{is} \cdot \frac{(L_i - L_{i0}) + |L_i - L_{i0}|}{2L_{i0}}$$

where F_{is} is the force per extensible foreign unit of each ligament, ΔL_i is the variation in the length of each ligament and L_{i0} is the zero foreign length of each ligament. This formulation generates a positive force only when the ligament is stretched beyond its own neutral length and zero when the ligament is relaxed.[4]

To represent the ligaments in the work of the authors Zheng, Fleisig, Escamilla and Barrentine (1998), spring-damper type elements are used, thus describing an analytical model in the sagittal plane with the aim of examining the forces of the joint during movement.

In the doctoral thesis supported by Mrs. Valerica Moşneguţu in 2008, the 3D model with 12 degrees of freedom was presented, resulting from the analysis of two remarkable works in the field - Shelburne, Pandy, Anderson and Torry (2004) and Wolf and Degani (2007).

The first paper develops a model with 18 degrees of freedom and combines theoretical and experimental aspects. The movement of the knee joint is modeled taking into account the movements of the entire human body, in this case the leg. The authors study and explain the nature of the forces transmitted by the ligaments to the knee joint. The most important aspects of the analysis in this paper are: a) the analysis of the movements of the human body, b) the analysis of the reaction forces of the ground, c) the analysis of the forces acting on the knee joint.

The disadvantage of this model is that the parameters and constants that appear in the modeling are assumed to be known a priori from experience. However, experience is not always able to directly provide these values by measurement, for their finding another model or an inverse problem is required.

The second work aims to develop an expert system for identifying knee pathology based on kinematic observations. A 6-degree-of-freedom model is described using screw theory and a screw-type representation of motion.

The degrees of freedom are represented as screw moment parameters (ISP), which are used to know the knee movements. The flexion of the joint with different pathologies is processed with an optical tracking system and then analyzed and diagnosed through the prism of this model. The optical information is composed of snapshots of the position and condition of the bones. ISP is calculated to know the movement from two successive sequences. It is observed that the movement is very varied, it is specific to each patient and it is possible to classify the movements in relation to different pathological deficiencies. Thus, specific clusters appear, larger or smaller, that reflect different pathologies, such as ACL/PCL deficiency, or walking deficiencies due to damage to the synovial fluid.[4]

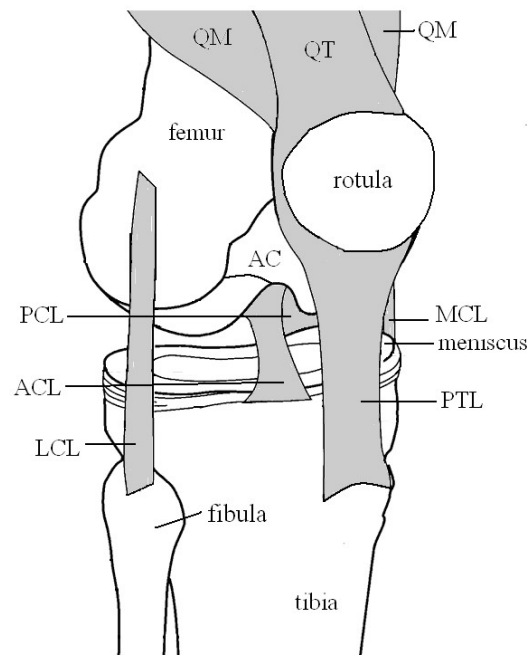


Figure 28

The elements of the knee joint considered for modeling are: femoral condyles, tibial plateau, AC articular cartilage, menisci, four ligaments and two tendons, namely: anterior cruciate ligament ACL, posterior cruciate ligament PCL, medial ligament MCL, lateral ligament LCL, quadriceps tendon QT and the PTL patellar tendon (figure 28). The articular surfaces are represented by the curved surfaces of the femoral condyles, the surfaces of the tibial coat and the facets of the patella. The AC articular cartilages serve as supporting surfaces, and the menisci as mobile supports. The muscles (quadriceps) in front of the femur, QM, straighten the knee (in extension), and the muscles behind the femur, the popliteus tendon, support the knee (in flexion). The patella works as a real lever for the quadriceps muscles, increasing their efficiency. The QT quadriceps tendon and the PTL patellar tendon enclose the patella, helping with its mechanical movements.

Mrs. Valerica Moşneguţu solves the following direct problem: *Given the forces in the muscles, tendons and ligaments, the internal-external rotation and flexion-extension angles of the knee, determine the movement of the patella relative to the femur during flexion and extension.* And inverse problem: *To determine the muscle forces, internal-external rotation and flexion-extension angles from the experimental data of the movement of the patella during flexion and extension.* In constructing the inverse problem, it is necessary both to model the movement of the patella and to model the geometric shape of the patella.

For the movement of the patella (jointly linked to the knee joint), build a model with 12 degrees of freedom. It introduces a global coordinate system for the knee joint and a local coordinate system for the femur, tibia and patella.

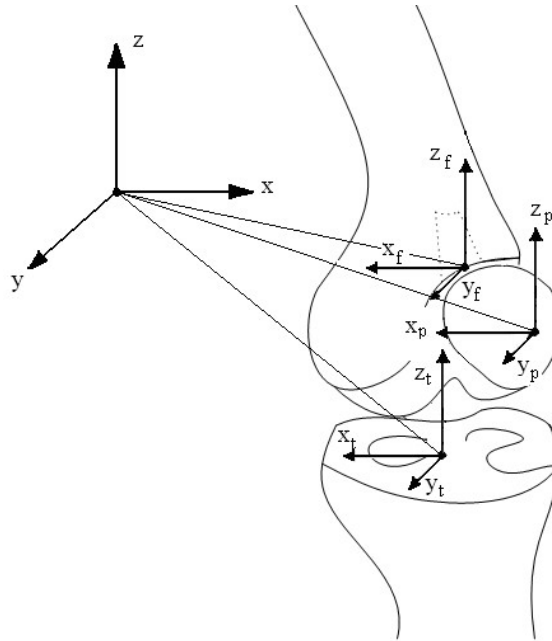


Figure 29

It is assumed that the position of the knee joint is described in relation to a global coordinate system and 3 local coordinate systems attached to each bone (figure 29): the femur (x_f, y_f, z_f) , the tibia (x_t, y_t, z_t) and, respectively, the patella (x_p, y_p, z_p) . The origins of the coordinate systems attached to the tibia and femur are in the centers of the intercondylar areas.

The origin of the patella coordinate system is located at the center of gravity of the patella. For all three systems, the axis x is oriented medially, the axis y , posterior, and the axis z , superior, parallel to the axes of the global system (x, y, z) . In the global coordinate system, the plane (x, y) is defined as the coronal plane and the plane (y, z) as the sagittal plane. In the global reference system, a point belonging to the articular surface, which we denote by Γ , is defined by the coordinates (x, y, z) , and in the local coordinate systems by (x'_L, y'_L, z'_L) , where the subscript L indicates the local system.

Transformation from system (x'_L, y'_L, z'_L) to system (x, y, z) is given by $x_i = u_i + R_{ik} x'_{kL}$, where R_{ij} is the rotation matrix $R = R(z, \varphi)R(x, \theta)R(z, \psi)$, with:

$$R(z, \psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

$$R(z, \varphi) = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where u is the translation vector.

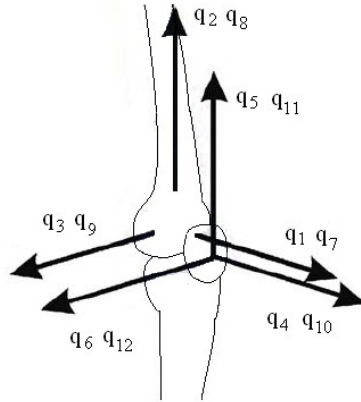


Figure 30

It is considered that the movement of the patella relative to the femur is described by 12 generalized coordinates (figure 30). The ball joint is considered as a massless body, and its motion is defined by a sequence of three rotations around the local axes. The displacement of the origin of the reference system attached to the patella compared to the origin of the reference system attached to the femur is defined by three translations along the three axes.

The movement of the knee joint relative to the global coordinate system attached to the pelvis is described by 12 generalized coordinates (degrees of freedom), namely: antero-posterior translation of the tibia relative to the femur q_1 , proximo-distal translation of the tibia relative to

the femur q_2 , medial-lateral translation of the the tibia in relation to the femur q_3 , the antero-posterior translation of the patella in relation to the femur q_4 , the proximo-distal translation of the patella in relation to the femur q_5 , the medial-lateral translation of the patella in relation to the femur q_6 ; varus-valgus rotation of the knee joint q_7 , internal-external rotation of the knee joint q_8 , flexion-extension rotation of the knee, q_9 patella q_{10} rotation, tilt rotation of the patella q_{11} and flexion-extension rotation of the patella q_{12} . Thus, the vector $q = \{q_1, q_2, q_3, \dots, q_{12}\}$ of generalized coordinates is obtained, of size 12×1 , coordinates that describe the configurations of the tibia-femur and patella-femur joints.

Under the assumption that the patella tendon is inextensible and that the penetration between the patella and the femur can be neglected, three holonomic constraints are defined for the movement of the patella relative to the femur, which can be combined with the six balance equations of the patellar moments and forces to provide a set of six algebraic equations for patellofemoral mechanics:

$$p_i(q, F_Q) = 0, \quad i = 1, 2, \dots, 6 \text{ or } \sum_{j=1}^{12} (\bar{a}_{kj} q_j + \bar{b}_j F_{mk} \delta_{jk}) = 0, \quad k = 1, 2, \dots, 6$$

with F_{mk} the dimension vector 6×1 , of the forces applied to the patella, from the QM and QT muscles, δ_{jk} is Kronecker's symbol, and the coefficients $\bar{a}_{kj}$ are $\bar{b}_j$ determined experimentally.

Realizing a one-to-one correspondence between a noted parameter t (time) and the angles of extension or flexion, the translation or rotation q depends on this parameter t .

The equations of motion of the knee joint can be written as follows

$$A(q)\ddot{q} + C(q, \dot{q}) + M_m(q)F_m + M_l(q)F_l + T(q, \dot{q}) = 0$$

where $A(q)$ is the dimension mass matrix 12×6 , $C(q, \dot{q})$ is a dimension vector 6×1 containing moments and centripetal and Coriolis forces resulting from the movement of the femur, F_m is a dimension vector of 6×1 the forces applied from the QM and QT muscles, $M_m(q)$ is a dimension matrix 12×6 describing the moment arms of the applied muscle forces, F_l is a dimension vector 6×1 containing the forces applied from the four ligaments and two tendons, $M_l(q)$ is a dimension matrix 12×6 describing the moment arms of the knee tendon and ligament forces, and $T(q, \dot{q})$ is a dimension vector of the 6×1 external moments applied to the knee joint.

The considered model does not take into account the fact that the joint of a normal knee is surrounded by the synovial membrane that contains cells that secrete the *synovial fluid*. The synovial fluid properties of a diseased knee vary considerably and are altered compared to those of a healthy knee (Chiroiu *et. al.*, 2010; Dowson, 1990). The experimental results highlighted an interesting phenomenon, namely the decrease of the viscosity coefficient of the synovial fluid in relation to the volume of the synovial fluid in the case of normal walking. The effect is similar to the Fahraesus and Lindquist effect observed experimentally in blood flow through small vessels (Munteanu, Chiroiu and Chiroiu, 2002; Munteanu, Donescu and Chiroiu, 2006).[4]

6 . CONCLUSIONS AND PERSPECTIVES

The elbow and knee joints are complex pendulum-type joints that can be modeled either as articulated bars or as systems made up of masses and springs.

The elbow joint is a trochlearthrosis with only one degree of freedom. It is made up of the humerus of the arm and the radius and ulna of the forearm. This joint allows only the execution of flexion and extension movements.

The knee joint consists of the lower end of the femur, the upper extremities of the tibia and fibula, and the patella. This joint has only one degree of freedom. The main movements of the knee are: flexion and extension of the calf on the thigh. Secondary movements are: internal rotation and external rotation.

For a proper modeling of them, they must be "broken" from the rest of the human body. Models are created to precisely accomplish a specific problem.

The next period of research will follow the validation of the theoretical results obtained at this level and through own experimental data. It will also aim to carry out a comparative study in relation to the normative documents regulating the negative and positive effects that vibrations have on the joints of the human body.

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